Reconstructing Dark Energy

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INSA Homi Jehangir Bhabha medal lecture, IISER, Pune, Dec 27, 2017

One of the most EXCITING observational discoveries of the past decade is that the

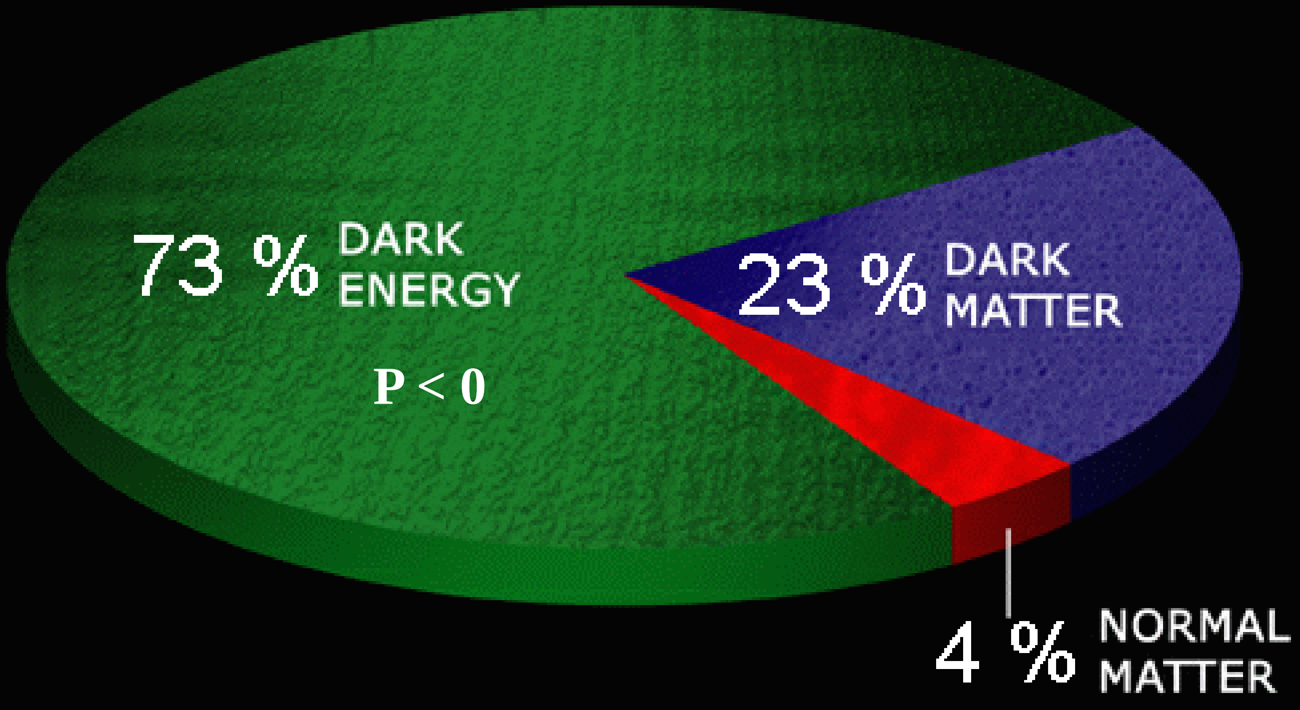
Universe is Accelerating !

The source responsible for Cosmic Acceleration is presently unknown is called

Dark Energy

Dark Energy has large negative pressure and could account for up to 70% of the total matter density in the Universe !!

Dark Energy has negative pressure and can make the Universe Accelerate !



Galaxies, stars, planets, tables, chairs, you, me….

- In Newtonian Gravity the gravitational potential is determined solely by the density of matter through the Poisson equation:

$$\nabla^2\phi = 4\pi G\rho$$

- In general relativity this is replaced by

Space-time curvature ⇨ $G_{ik} = 8\pi G T_{ik}$ ⇨ **Energy-momentum tensor**

``Matter tells space how to curve,
Space tells matter how to move'' *J.A. Wheeler*

In a homogeneous and isotropic setting $T_i^k = diag(\rho, -P, -P, -P)$

and the Poisson equation changes to $\nabla^2\phi = 4\pi G(\rho + 3P)$

In other words `pressure carries weight' in Einstein's gravity.
The dynamics of space-time is influenced not only by the density of matter but also by its PRESSURE !

If $\rho + 3P < 0$

then gravity becomes REPULSIVE and the Universe can **Accelerate** !!

The `simplest' candidate for

Dark Energy

is

The Cosmological Constant: `Λ'.

Introduced by Einstein in 1917, the cosmological constant satisfies

$$T_i^k = \Lambda \delta_i^k$$

Which implies $P = -\rho \quad (\rho = \Lambda)$

for the equation of state of the cosmological constant.

The year 2017 marks 100 years since the inception of the Cosmological Constant !

Brief History

- In 1917 Einstein proposes the Cosmological Constant `Λ' and constructs a closed quasi-static universe using:

$$G_{ik} = 8\pi G\, T_{ik} + \Lambda g_{ik}$$

In a letter to Ehrenfest Einstein writes

``I have again perpetrated something relating to the theory of gravitation that might endanger me of being committed to a madhouse''.

- In 1917 de Sitter presents vacuum solutions of $G_{ik} = \Lambda g_{ik}$

- In 1922 Alexander Friedman constructs a matter dominated expanding universe without Λ.

- In 1923, in a letter to Weyl, Einstein says

``If there is no quasi-static world, then away with the cosmological term ! ''

- *Much later, when I was discussing cosmological problems with Einstein, he remarked that the introduction of the cosmological term was the biggest blunder of his life. -- George Gamow, My World Line, 1970*

THE COSMOLOGICAL CONSTANT AND THE THEORY OF ELEMENTARY PARTICLES

Ya. B. ZEL'DOVICH

Institute of Applied Mathematics, USSR Academy of Sciences

Usp. Fiz. Nauk 95, 209–230 (May, 1968)

Interest in gravitation theory with a cosmological constant was revived in 1967. Three papers were published, by Petrosian, Salpeter, and Szekeres in the USA[1] and by Shklovskii[2] and Kardashev[3] in the USSR, in which evolutionary universe models[1] in such a theory (the Λ models) are considered. The stimulus for the revival of the theory was provided by new observational data on remote quasistellar sources (quasars and quasags, QSR and QSG in the English-language literature).* It turned out, first of all, that for these objects the connection between the brightness and the red shift does not fit the simple models without a cosmological constant (and without assumptions concerning the evolution of the quasars!). In addition, as noted by the Burbidges[4], in ten quasars whose spectra have revealed absorption lines the red shift of these lines $z = (\lambda - \lambda_0)/\lambda_0$ lies in the narrow range $1.94 < z < 1.96$ or even $1.945 < z < 1.955$. This phenomenon will henceforth be referred to briefly as $z = 1.95$.

The Λ models were introduced in[1] to explain the observed relation between the red shift and the brightness; the explanation of $z = 1.95$ in the absorption spectrum was touched upon casually. References 2 and 3 are devoted entirely to the explanation of $z = 1.95$: the absorption lines are ascribed to galaxies lying along the path of the light ray arriving from the quasar. The predominant appearance of one value of z is attributed by the authors to the fact that with this z[2] the expansion of the universe was greatly slowed down both compared with the preceding period ($z > 1.95$) and compared with the succeeding period ($z < 1.95$ up to $z = 0$, corresponding to the present time). The slowed-down expansion leads to an increase of the path traversed by the ray in the corresponding interval of z, and increases the probability that the quasar light ray will encounter a galaxy and that absorption lines with precisely this value of z, i.e. about 1.95, will be imprinted in it.[3]

An expansion law with a sharp deceleration at a definite value of z is possible only for the Λ models; it is necessary here to satisfy with great accuracy the relation between the total amount of matter in the universe and the value of the cosmological constant Λ. The discussed model is closed in its three dimensional geometrical structure. As shown by Kardashev[3], the assumption

The cosmological constant was placed on a firm physical foundation by Zeldovich who showed that

$$\langle T_{ik} \rangle_{\rm vac} = \Lambda g_{ik}$$

ie. the vacuum had properties reminiscent of a Λ term !

Prescient statement which paved the way for future advances including Inflation (1980's) and Dark energy (2000).

Both require $P < 0$!

``The Genie [cosmological constant] has been let out of the bottle, and it is no longer possible to force it back in''. – Zeldovich (1968)

Energy of the quantum vacuum

Observed:	$\rho \simeq 10^{-30}$	$\mathrm{g\ cm^{-3}}$
Quantum field theory:	$\rho = \infty$	$\mathrm{g\ cm^{-3}}$
Quantum gravity:	$\rho \approx 10^{+90}$	$\mathrm{g\ cm^{-3}}$
Supersymmetry:	$\rho \approx 10^{+30}$	$\mathrm{g\ cm^{-3}}$
Higgs potential:	$\rho \approx -10^{+25}$	$\mathrm{g\ cm^{-3}}$
Other sources:	$\rho \approx \pm 10^{+20}$	$\mathrm{g\ cm^{-3}}$

The Cosmological Constant Problem

As I was going up the stair
I met a man who wasn't there
He wasn't there again today
I wish, I wish he'd stay away

Hughes Mearns

SN

pre-1998 – why is $\Lambda = 0$?

post-1998 – why is Λ so small ?

$$\rho_\Lambda = \frac{\Lambda}{8\pi G} \simeq 10^{-47} GeV^4 \ ?!$$

Nice review of cosmological constant problem: S. Weinberg, Rev.Mod. Phys. 1989

ASTRONOMY

Exploding Stars Point to a Universal Repulsive Force

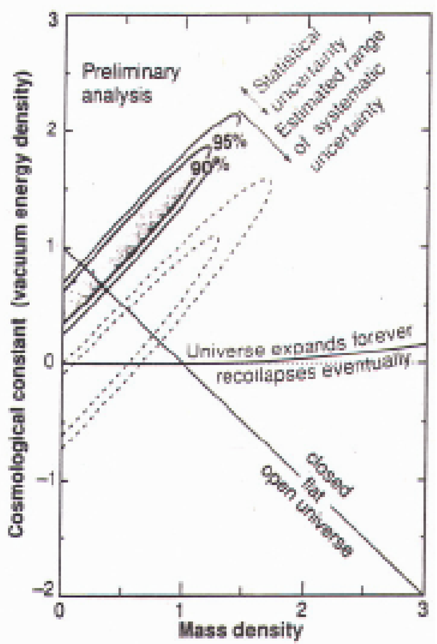
By now, even newspaper readers with a casual interest in astronomy may have heard the unsettling message delivered by distant, exploding stars called supernovae: The universe will likely expand infinitely, growing ever more tenuous. Now a new batch of supernovae has lent support to a strange picture of just what the universe is made of. A preliminary analysis may provide the first strong evidence that the universe could be permeated by a large-scale repulsive force. The reservoir of energy fueling that force could be anything from a quantum-mechanical shimmer in empty space, called the cosmological constant, to even more exotic possibilities that go by names like X-matter and quintessence.

At the meeting of the American Astronomical Society in Washington, D.C., earlier this month, Saul Perlmutter of Lawrence Berkeley National Laboratory in Berkeley, California, announced that he and an international team of observers have now studied a total of 40 far-off supernovae, using them as beacons to judge how the cosmic expansion rate has changed over time. Not only did the results support the earlier evidence that the expansion rate has slowed too little for gravity ever to bring it to a stop; they also hinted that something is nudging the expansion along. If they hold up, says Perlmutter, "that would introduce important evidence that there is a cosmological constant."

"It would be a magical discovery," adds Michael Turner of the University of Chicago. "What it means is that there is some form of energy we don't understand." Other observers had already found signs that the universe contains far less mass than the mainstream theory of the big bang predicts, which left open the possibility that some form of energy in empty space could be making up the deficit. The cosmological constant—also called lambda—is a longtime candidate for serving as this energy reservoir. But the new

was sparked when a fleck of the primordial vacuum underwent a chance fluctuation that filled it with something much like a colossally intense cosmological constant. This "scalar," or directionless, field drove the patch into an exponential growth spurt. As the patch expanded and cooled, energy from the scalar field fed an explosion of material particles: The material universe was born—"creating everything from nothing," as the theory's creator, Alan Guth of the Massachusetts Institute of Technology, puts it.

During the exponential growth spurt, inflation would have ironed out any primordial



What the stars show. A preliminary analysis of 40 distant supernovae, reported by the Supernova Cosmology Project, offers strong evidence for an energy density in empty space, if space is "flat." The green regions indicate statistical uncertainties; the dashed lines show the preliminary estimates (now being refined) if all the systematic uncertainties added up in one direction.

Dawn of Dark Energy

Based on observations of distant Type Ia Supernovae.

Science 30 January 1998

[Based on Perlmutter, et al., Ap J (1998); also see Riess, et. al. Astron. J (1998)]

Perlmutter, Riess and Schmidt were awarded the 2011 Nobel prize for this discovery.

In flat Euclidean space the flux of light from a distant source is

$$\mathcal{F} = \frac{\mathcal{L}}{4\pi r^2}$$

supernovae

BUT in an expanding universe: $$\mathcal{F} = \frac{\mathcal{L}}{4\pi d_L^2}$$

where $$d_L(z) = (1+z)\int_0^z \frac{dz'}{H(z')}$$ Is the luminosity distance,

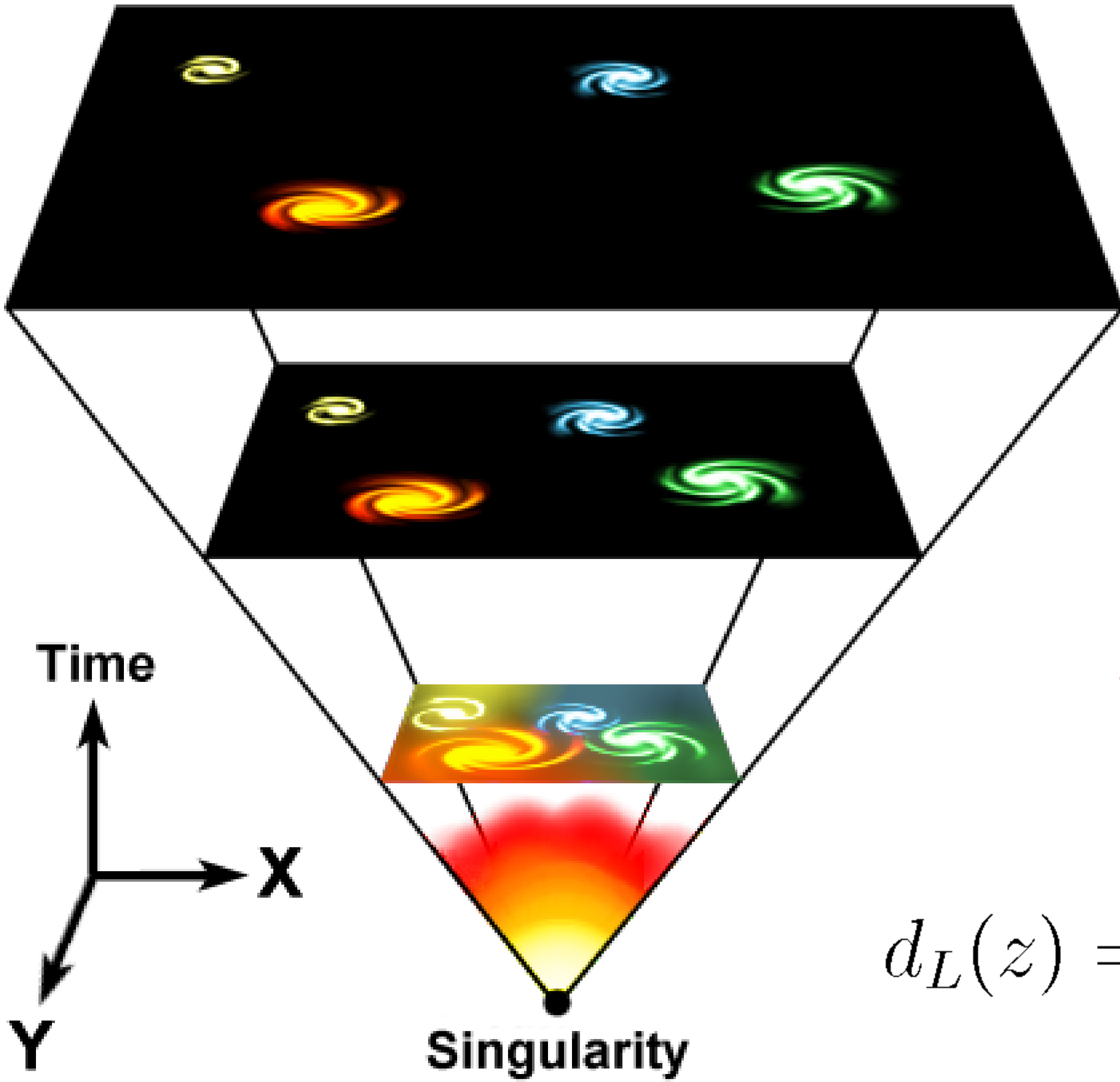
Dark matter
+ dark energy

and $$H^2(z) = \frac{8\pi G}{3}[\rho_{\text{baryons}} + \rho_{\text{radiation}} + \rho_{\text{DM}} + \rho_{\text{DE}}]$$

is the expansion rate: V = HR. The light source is at redshift $z = \frac{R_0}{R(t)} - 1$

and R(t) is the expansion factor of the universe.

Thus the dimming of light by a distant supernova is caused by the total energy density of the universe including: baryons, leptons, dark matter and dark energy.



V = HR

Hubble's law

$$\mathcal{F} = \frac{\mathcal{L}}{4\pi d_L^2}$$

$$d_L(z) = (1+z)\int_0^z \frac{dz'}{H(z')}$$

$$H^2(z) = \frac{8\pi G}{3}\left[\rho_{\rm baryons} + \rho_{\rm radiation} + \rho_{\rm DM} + \rho_{\rm DE}\right]$$

The `simplest' candidate for
Dark Energy
is
The Cosmological Constant: `Λ'.

Introduced by Einstein in 1917, the cosmological constant satisfies

$$T_i^k = \Lambda \delta_i^k$$

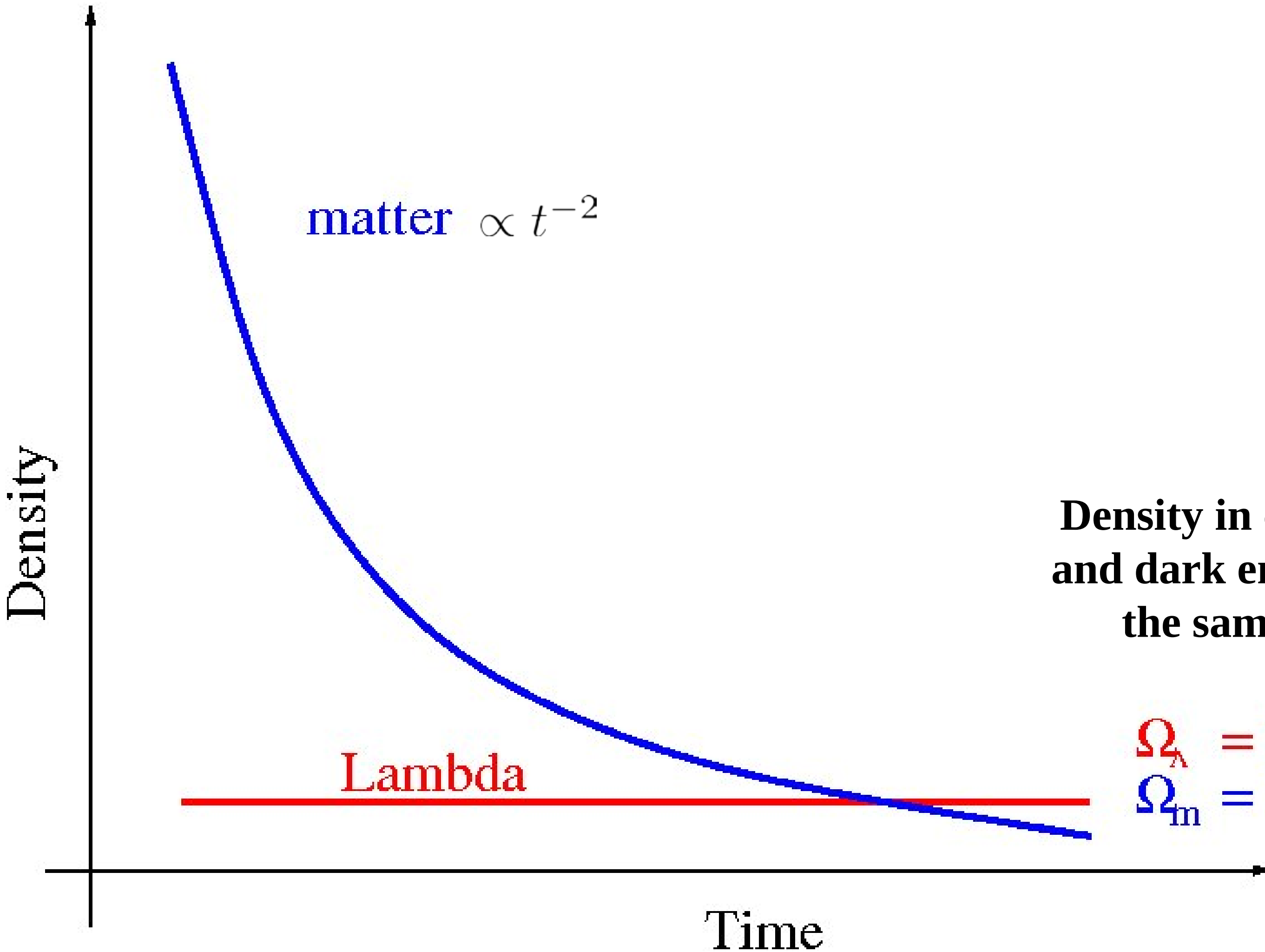
Which implies $P = -\rho \quad (\rho = \Lambda)$

for the equation of state of the cosmological constant.

The density in the cosmological constant remains constant as the universe evolves, while the density in other forms of matter/radiation decreases.
Therefore the cosmological constant dominates the universe at late times !

$\rho_\Lambda/\rho_m \sim 10^{-44}$ at the electroweak scale if $\rho_{0m} \sim \rho_\Lambda$ now.

Initial conditions need fine tuning ?



Density in dark matter and dark energy almost the same today !

$\Omega_\Lambda = 2/3$

$\Omega_m = 1/3$

Cosmic coincidence ?

The dilemma of a cosmological constant prompted researchers to look for dynamical models of Dark Energy.

Dynamical Dark Energy

1. Quintessence. One models dark energy using a scalar field :

$$\mathcal{L} = \partial_\mu \phi \partial^\mu \phi - V(\phi) \ , \ T_{ik} = \phi_{,i}\phi_{,k} - g_{ik}\mathcal{L}$$

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi), \ P = \frac{1}{2}\dot{\phi}^2 - V(\phi).$$ Where $-1 \leq w \equiv P/\rho \leq 1$

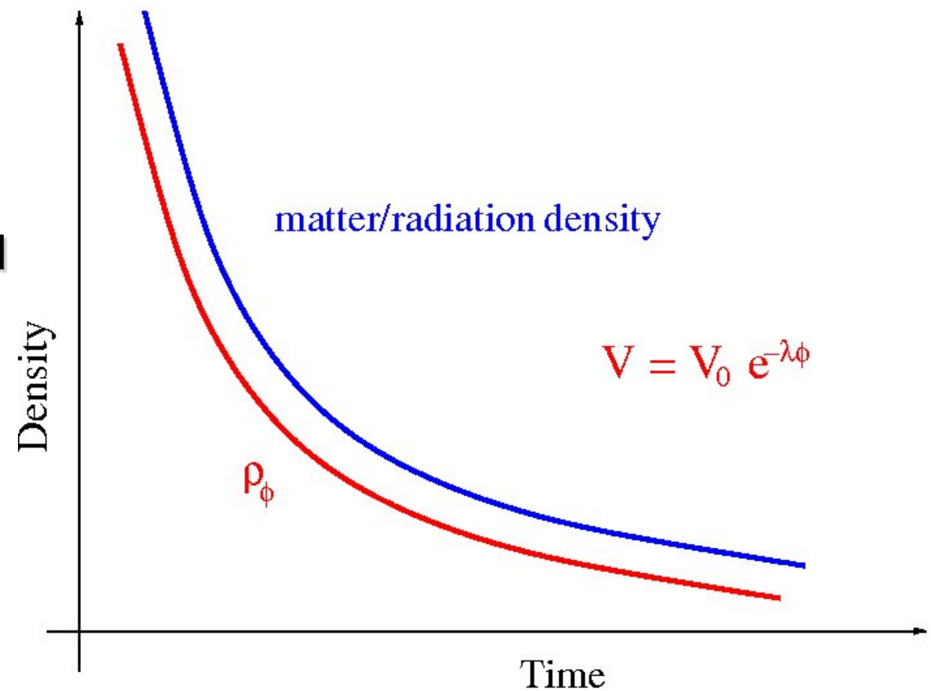
If $V''V/V'^2 \geq 1$ the scalar tracks the dominant matter component.

The exponential potential $V(\phi) = V_0 \exp(-\lambda\phi/M_P)$ leads to tracker behavior

$$\frac{\rho_\phi}{\rho_{\rm total}} = \frac{3(1+w_B)}{\lambda^2} = constant$$

where w_B is the EOS of the background

and $M_P = 1/\sqrt{8\pi G}$.



$V \propto \phi^{-\alpha}, \ \alpha > 0$

Ratra and Peebles (1988)
Wetterich (1988)
Ferreira and Joyce (1998)

2. Chaplygin Gas

The Born-Infeld lagrangian density $$\mathcal{L} = -V_0\sqrt{1-\phi_{,\mu}\phi^{,\mu}}$$

leads to the Chaplygin gas $(A = V_0^2)$ $$p = -\frac{A}{\rho}$$ < 0 !

The conservation equation

$$dE = -pdV \Rightarrow d(\rho a^3) = -pd(a^3)$$ gives $$\rho = \sqrt{A + \frac{B}{a^6}}$$

(B is a constant of integration.)

So that $\rho \propto a^{-3}$ at early times (like matter)

while $\rho \longrightarrow \mathrm{constant}$ at late times -- just like Λ !!

The Chaplygin gas behaves like pressureless matter at early times and like a cosmological constant during late times !!

Q. Can Chaplygin gas unify dark matter and dark energy ?

[Kamenshchik, Moschella, & Pasquier (2001)]

NO

Numerous Dark Energy models have been suggested to account for an accelerating Universe:

(i) Cosmological constant

(ii) Quiescence with w = constant < -1/3, (cosmic strings/walls),

(iii) Quintessence models;
(iv) The Chaplygin gas;
(v) Phantom DE (w < -1);
(vi) Oscillating DE;
(vii) Models with interactions between DE and dark matter;
(viii) Scalar-tensor DE models;
(ix) Modified gravity models:
(x) Dark energy driven by quantum effects,
(xi) Higher dimensional braneworld models,
(x) Galileon models, etc.

Faced with the increasing proliferation of DE models a cosmologist can proceed in two ways:

(i) Test each and every model against observations.
(ii) Reconstruct properties of dark energy in a **model independent** manner.

Saini, Raychaudhury, Sahni and Starobinsky, PRL 85, 1162 (2000)

Of all Dark Energy models the cosmological constant is singled out by its elegance and simplicity:

$$T_i^k = \Lambda \delta_i^k \ .$$

So, as a first step, its logical to find tests which could falsify

$$\Lambda\mathrm{CDM}$$

$$\Downarrow$$

NULL tests for the cosmological constant Λ

$\Lambda\mathrm{CDM}$

$\Lambda + \mathrm{Cold\ Dark\ Matter\ (CDM)}$

also known as **LCDM**

`standard model' of cosmology

[VS, Saini, Starobinsky, Alam JETP 2003; VS, Shafieloo, Starobinsky PRD 2008, Zunckel & Clarkson PRL 2008, Arabsalmani & VS, PRD 2011, VS, Shafieloo, Starobinsky, ApJLett 2014, etc.]

Model independent reconstruction of Dark Energy:

The expansion factor of the universe $a(t)$ provides the most general information about expansion history:

present time

$$a(t) = a(t_0) + \dot{a}\big|_0 (t - t_0) + \frac{\ddot{a}\big|_0}{2}(t - t_0)^2 + \frac{\dddot{a}\big|_0}{6}(t - t_0)^3 + \cdots$$

In 1970 Alan Sandage described observational cosmology as being ``a search for two numbers": $H_0 = (\dot{a}/a)_0 \quad q_0 = -(\ddot{a}/aH^2)_0$.

In this era of `precision cosmology' let us define a third number $r = \frac{\dddot{a}}{aH^3}$

Surprisingly, $r = 1$ only in LCDM !

For all other dark energy models $r \neq 1$

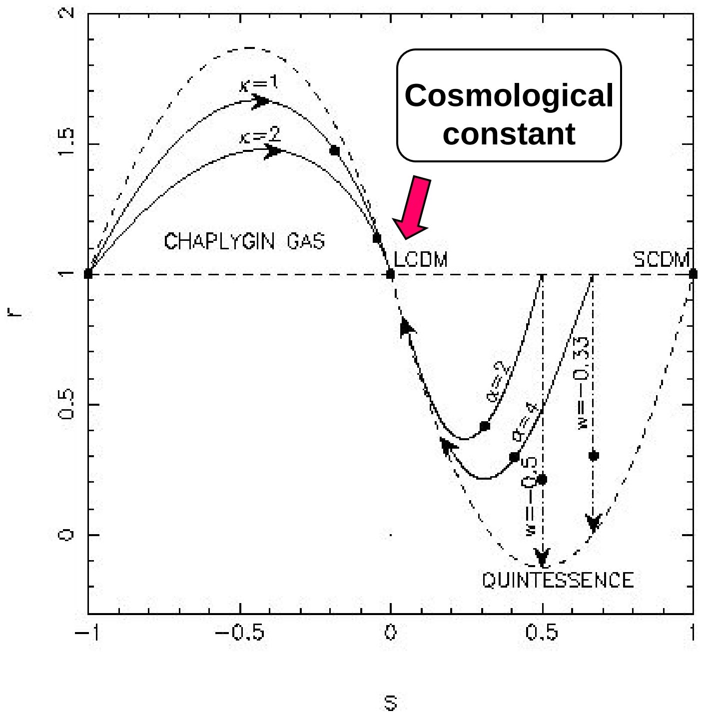
Similarly define $s = \frac{r-1}{3(q-1/2)}$. s = 0 only in LCDM !

Therefore {r,s} are null diagnostics for the cosmological constant, since

$\{r, s\} = \{1, 0\}$ only for Λ

The Statefinder pair {r,s} is an excellent diagnostic of Dark Energy !

[Sahni, Saini, Starobinsky, Alam (2003)]



$$r = \frac{\dddot{a}}{aH^3}\ , \quad s = \frac{r-1}{3(q-1/2)}$$

r = 1, s = 0: fixed point for the cosmological constant !

Quintessence: $V(\phi) \propto \phi^{-\alpha}$

Statefinder provides a fingerprint of Dark Energy !

It can easily distinguish the cosmological constant from other models.

Interestingly, null tests of LCDM can also be constructed from higher derivatives of the expansion factor.

[Arabsalmani & Sahni (2011)]

Practically the Statefinders can be determined from observations of

Standard Candles (type Ia Supernovae) and Standard Rulers (BAO)

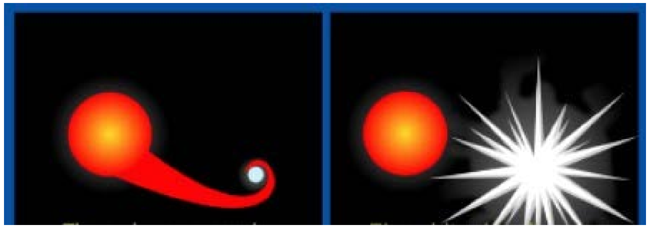
$$\mathcal{F} = \frac{\mathcal{L}}{4\pi d_L^2} \qquad d_L(z) = (1+z)\int_0^z \frac{dz'}{H(z')}$$

The observed quantity d_L needs to be differentiated thrice to determine the Statefinder

Since $r = \frac{\dddot{a}}{aH^3}$; and $H = \dot{a}/a$.

But this is a noisy operation, since errors increase on differentiating a noisy quantity -- d_L

Also type Ia supernovae involve unknown systematics !

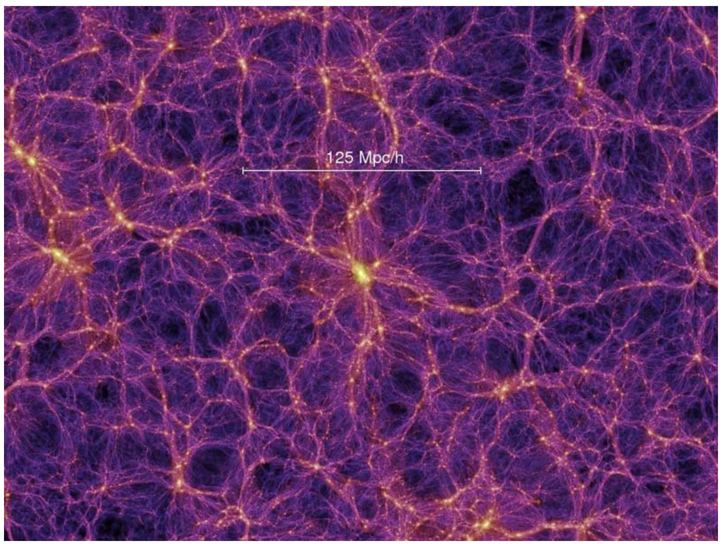


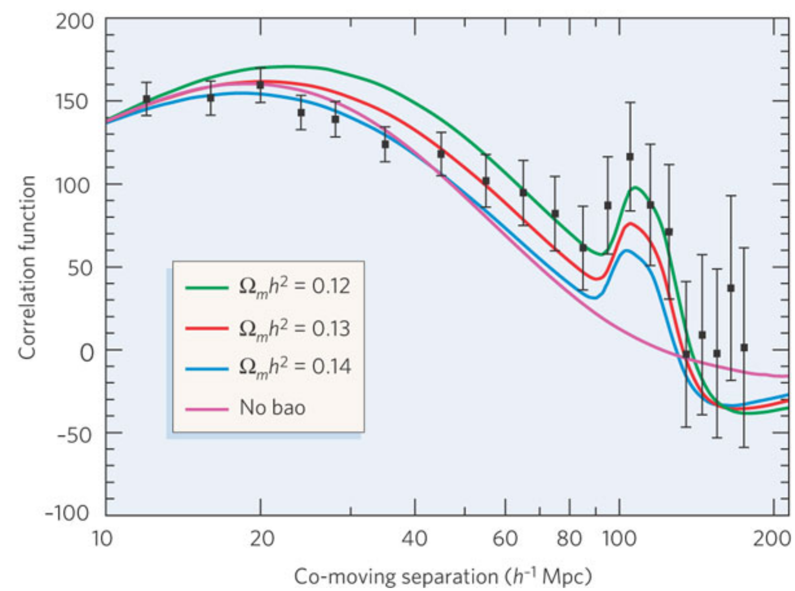
Highly non-linear system, difficult to simulate in the laboratory.

Baryon Acoustic Oscillations (BAO)

The galaxy distribution contains an imprint of the primordial fluctuations in the photon-baryon plasma. Prior to photon decoupling $(z \sim 1100)$ gravity creates oscillations in the photon-baryon plasma. After decoupling these oscillations correspond to a characteristic scale $\sim 150 Mpc$ (comoving horizon at recombination). This scale behaves like a standard ruler and can be used to determine the nature of DE.

Sunyaev & Zeldovich (1970) *Peebles & Yu (1970)*





Large length scale, better understood systematics.

Nature 440, 1126 (2006)

Galaxy clustering is anisotropic and the BAO scale can be measured both in the radial and the transverse direction. Radial direction gives $H = \dot{a}/a.$

H needs to be differentiated twice to get the Statefinder parameter $r = \dfrac{\dddot{a}}{aH^3}$

But differentations amplify noise, so

Can one determine a null diagnostic only from H(z) ?

(with no differentiations)

$$\text{Expansion history} = \frac{\dot{R}}{R}$$

V = HR

Yes, its called the OM diagnostic.

[Sahni, Shafieloo and Starobinsky, PRD, 2008]

The Om diagnostic – a null test for the Cosmological Constant.

$$Om(z) = \frac{h^2(z) - 1}{(1+z)^3 - 1} \qquad h(z) = \frac{H(z)}{H_0}, \;\; H = \frac{\dot{a}}{a}$$

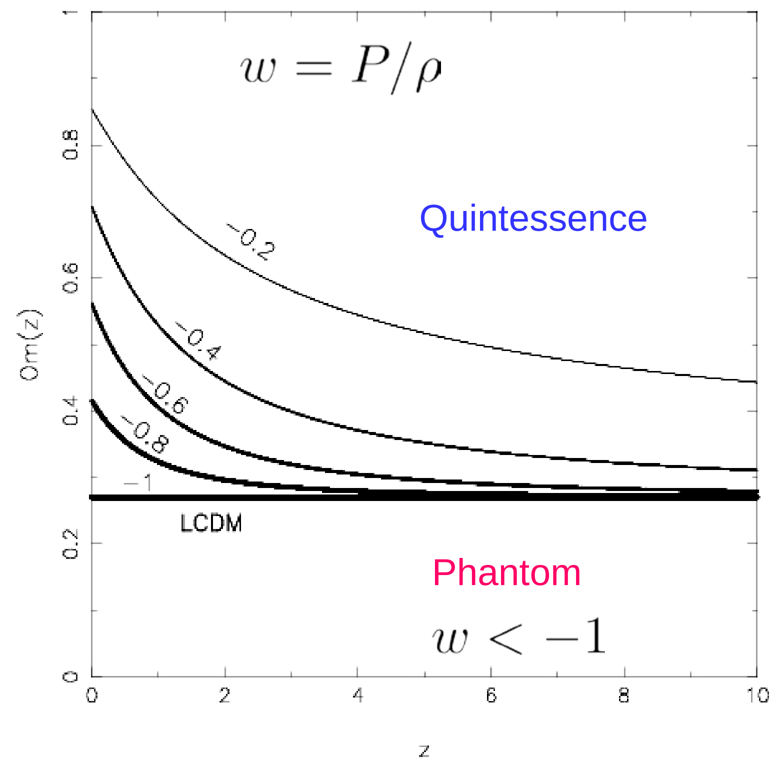
Om is constant only for the Cosmological Constant !

For all other Dark Energy models Om evolves with time.

$$Om(z) = \Omega_{0m} \quad \text{for } \Lambda \;.$$

$$Om(z) > \Omega_{0m} \text{ in Quintessence}$$

$$Om(z) < \Omega_{0m} \text{ in Phantom}$$



So if Om evolves with redshift then the Cosmological constant is ruled out !

[Sahni, Shafieloo, Starobinsky PRD 2008]

But one can do even better by defining the two-point diagnostic:

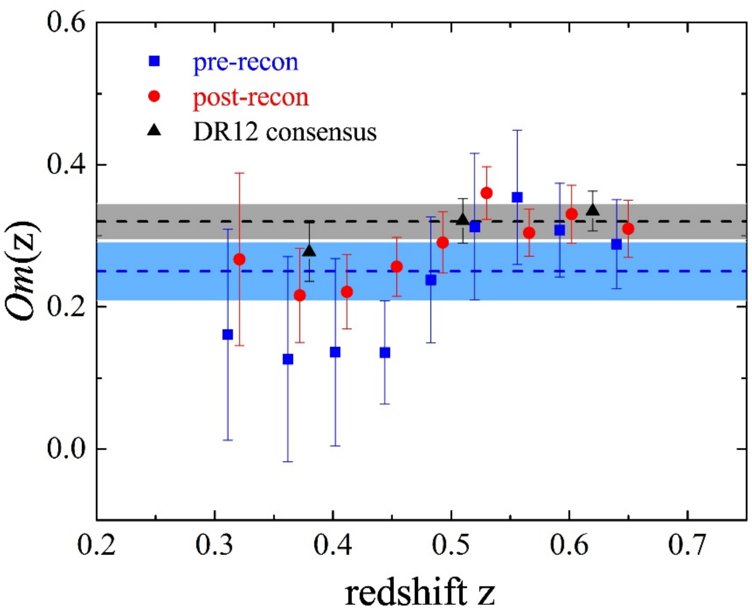
$$Omh^2(z_i, z_j) = \frac{h^2(z_i) - h^2(z_j)}{(1+z_i)^3 - (1+z_j)^3}, \text{ where } h(z) = \frac{H(z)}{100\text{km/sec/Mpc}}$$

Advantage: For the cosmological constant: $\underbrace{Omh^2 = \Omega_{0m}h^2 = 0.1426 \pm 0.0025}$

CMB

Future observations will allow us to determine **Om** from Euclid and SKA data to sub-percent accuracy !

[VS, Shafieloo, Starobinsky, ApJLett 2014]



H(z) is currently known at 10 redshifts, 9 of which are at low z, and have been used to determine Om by the SDSS team.

Results are consistent with a cosmological constant. Wang et. al. arXiv:1607.03154

Two dark clouds on the horizon of the cosmological constant.

- At early times the universe seems to expand slower than with a cosmological constant.
- The presence of galaxies and quasars at high $z > 10$ might suggest an age problem.

Although low redshift data is consistent with Λ high redshift data may be **problematic** for the cosmological constant.

The value obtained by Delubac et al: $H(z=2.34) = 222 \pm 7 \text{ km/sec/Mpc}$

is much lower than the LCDM value $H(z=2.34) = 238 \text{ km/sec/Mpc}$

This can happen in models in which the cosmological constant is screened

$$H^2(z) = \underbrace{\frac{\Lambda}{3} - f(z)}_{\Lambda_{\rm eff}/3} + \kappa\rho_{0m}(1+z)^3, \quad f(z) > 0$$

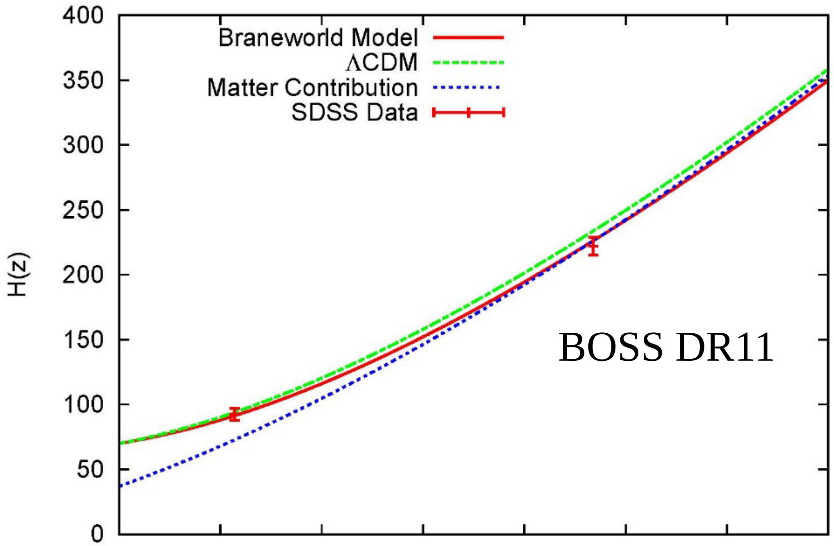
$\Lambda_{\rm eff}/3 \Rightarrow \Lambda_{\rm eff}$ **grows with time !**

This can happen in modified gravity models including Braneworld models

[VS & Shtanov 2003]

Or if there is interaction between dark matter and dark energy

[Amendola 2000, Abdalla et al. 2014, Shafieloo, Hazra, Sahni, Starobinsky 2017]



Two ways of making the Universe ACCELERATE:

- modify the MATTER sector:

$$G_{\mu\nu} = 8\pi G\, T_{\mu\nu}$$

This leads to Physical models of DE such as Quintessence, Chaplygin Gas, Phantom matter etc.

- modify the GRAVITY sector:

$$G_{\mu\nu} = 8\pi G\, T_{\mu\nu}$$

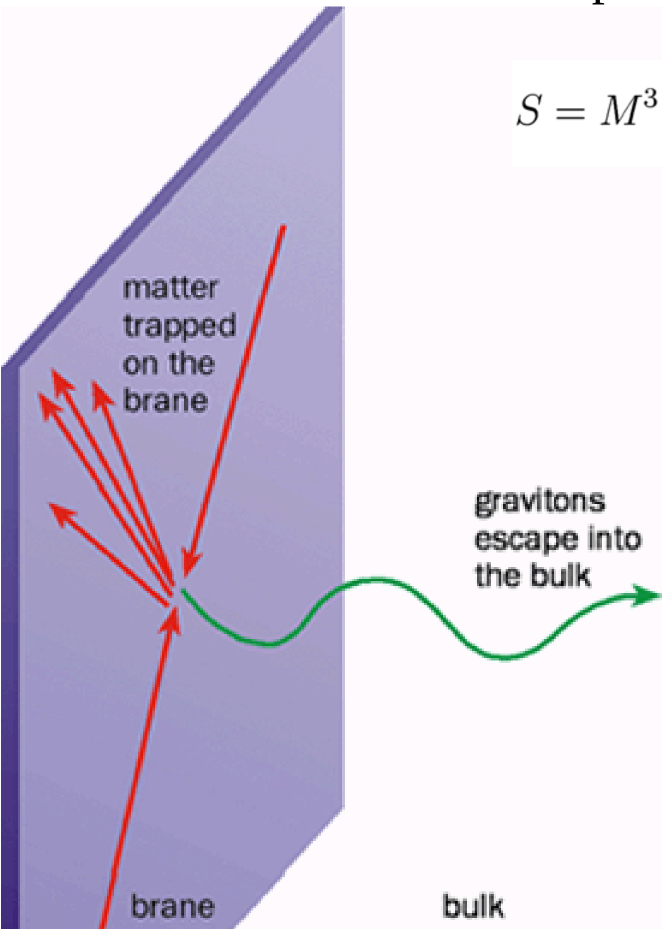
The cosmological constant introduced by Einstein in 1917 was the first model of this kind since

$$G_{\mu\nu} + \lambda g_{\mu\nu} = 8\pi G\, T_{\mu\nu}$$

This leads to Geometrical models of DE such as higher dimensional (Braneworld) Gravity, scalar-tensor gravity, string/M-theory inspired models, f(R) gravity, etc.

One possible way of achieving a phantom equation of state $w_{\rm eff} < -1$ without running into instabilities is in models in which dark energy is screened.

An example is provided by Braneworld models (Sahni & Shtanov 2003).



$$S = M^3\left[\int_{\rm bulk}(\mathcal{R} - 2\Lambda_{\rm b}) - 2\int_{\rm brane} K\right] + \int_{\rm brane}(m^2R - 2\sigma) + \int_{\rm brane} L\,(h_{ab},\phi)$$

(Includes GR, DGP and Randall-Sundrum as subclasses)

$$h^2 = \Omega_{0m}(1+z)^3 + \underbrace{\Omega_\Lambda - f(z)}_{\text{Screened DE}}$$

$$f(z) = 2\sqrt{\Omega_\ell}\sqrt{\Omega_{0m}(1+z)^3 + \Omega_\sigma + \Omega_\ell}$$

Screening term, whose value increases with redshift !

$\Rightarrow$ $w_{\rm eff} < -1$

$w_{\rm eff} = P/\rho$

$$\Omega_\sigma = \frac{\sigma}{3m^2H_0^2}\,,\ \Omega_\ell = \frac{1}{\ell^2 H_0^2}$$

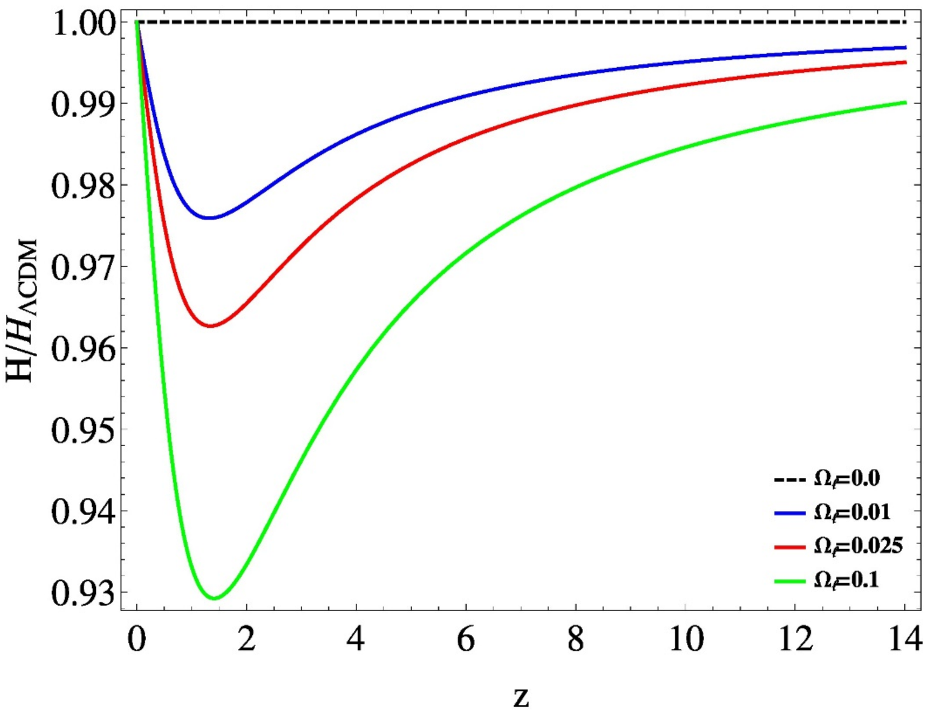
where $\Omega_\Lambda = \Omega_\sigma + 2\Omega_\ell$

$\ell = 2m^2/M^3$ is a new length scale !

Two properties of Screened Dark Energy

$$H^2(z) = \frac{\Lambda}{3} - f(z) + \kappa\rho_{0m}(1+z)^3, \quad f(z) > 0$$

H(z) is lower than in LCDM. This will affect cosmological quantities.



Luminosity distance: D_L increases

$$\frac{D_L(z)}{1+z} = \int_0^z \frac{dz'}{H(z')}$$

$$F = \frac{\mathcal{L}}{4\pi D_L^2}$$

decreases !

- SNIa are fainter than in LCDM.

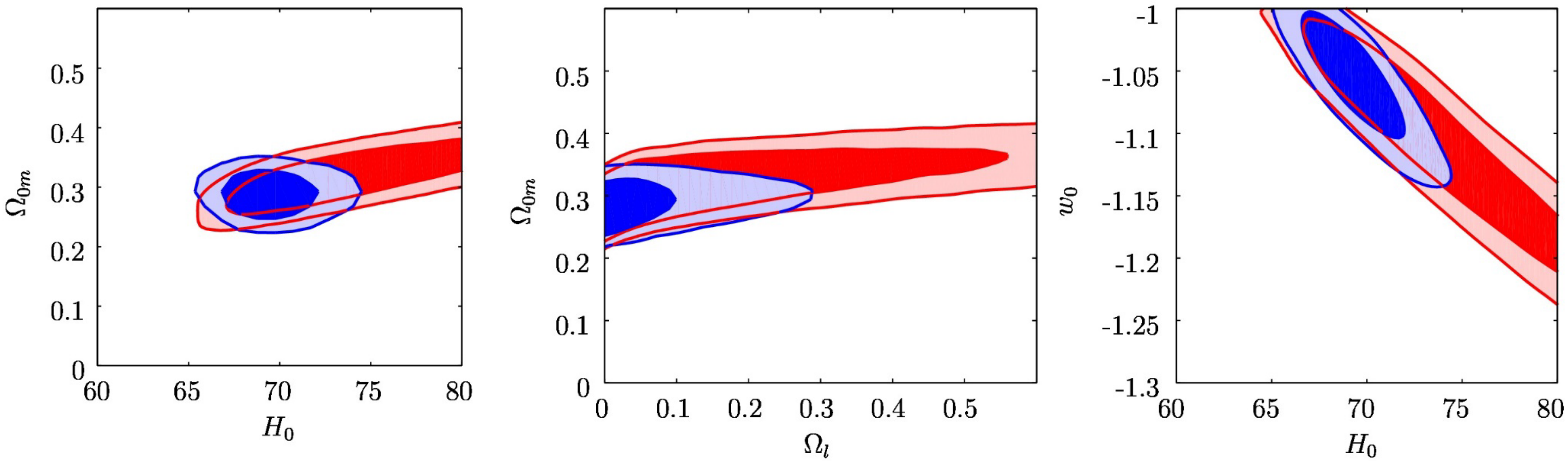
Age of the Universe:

$$T(z) = \int_z^\infty \frac{dz'}{(1+z')H(z')}$$

- Universe containing Screened DE is older than LCDM at a given redshift.

- Will also affect angular size distance, optical depth to electron scattering, AP test, etc.

Observational constraints on the phantom brane



Red contours show 1 and 2σ CL's for SNe+BAO data, while blue contours show results for SNe+BAO+CMB.

$w_0 < -1$ is preferred

Larger values of H_0 prefer a more negative equation of state for dark energy.

Alam, Bag & VS, PRD 2017 [arXiv:1605.04707]

Large values $H_0 = 73.24 \pm 1.74 \ km/s/Mpc$ obtained from HST observations by Riess et al., **arXiv:1604.01424**

$$w \equiv p_{DE}/\rho_{DE} = \frac{2q(x)-1}{3(1-\Omega_m(x))} \equiv \frac{\frac{2x}{3}\frac{d\log H}{dx} - 1}{1-(H_0/H)^2\Omega_{0m}x^3} \ , \ x = 1+z \ .$$

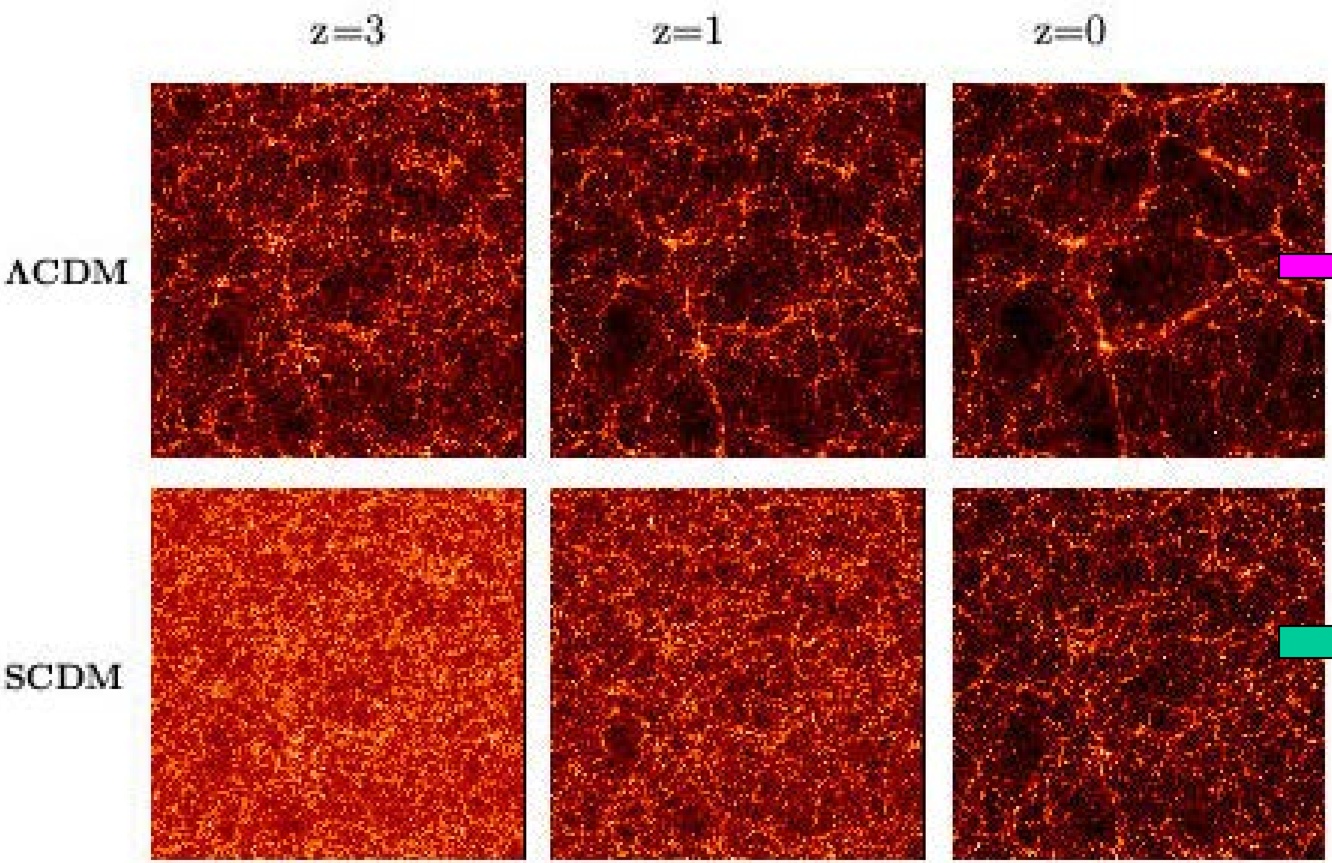
Do modified gravity models including Braneworlds have other properties which could distinguish them from the cosmological constant and similar DE models (Quintessence, etc.) ?

YES. Perturbations in modified gravity models grow at a different rate !

Q. How to break the degeneracy between two models Which have the same expansion history ?

Ans. The Cosmic Web comes to our rescue: the evolution of the Web will be different in $\Lambda\mathrm{CDM}$ and the Braneworld !

The evolution of the cosmic web is a sensitive probe of Dark Energy !



$$\ddot{\Delta}_m + 2H\dot{\Delta}_m \approx \frac{\rho_m \Delta_m}{2m^2}$$

$$\Omega_m \simeq 0.3, \Omega_\Lambda \simeq 0.7$$

Dark energy + matter

$\Lambda\mathrm{CDM}$

$$\Omega_m = 1, \Omega_\Lambda = 0$$

No dark energy,
Only dark matter.

Perturbation equations on the brane:

Viznyuk, Shtanov and Sahni, PRD 89, 083523 (2014)

$$ds^2 = -(1+2\Phi)dt^2 + a^2(1-2\Psi)\gamma_{ij}dx^i dx^j$$

$\Psi = \Phi$ in ΛCDM, but $\Psi \neq \Phi$ on the brane.

$$\dot{\Delta}_m = -\frac{k^2}{a^2}V_m + \frac{2(2+\beta)\rho_r}{m^2\beta}(V_r - V_m) + \frac{4\rho_r}{m^2\beta}V_{\mathcal{C}}\,, \tag{1}$$

$$\dot{V}_m = \frac{(2-\beta)a^2}{2m^2k^2\beta}\left(\rho_m\Delta_m + \frac{4\rho_r}{3}\Delta_r\right) + \frac{4\rho_r a^2}{3m^2k^2\beta}\delta_{\mathcal{C}} + \frac{4(2-\beta)\rho_r a^2}{m^2k^2\beta(2+\beta)}HV_{\mathcal{C}}\,, \tag{2}$$

$$\dot{\Delta}_r = H\Delta_r - \frac{k^2}{a^2}V_r + \frac{3(2+\beta)\rho_m}{2m^2\beta}(V_m - V_r) + \frac{4\rho_r}{m^2\beta}V_{\mathcal{C}}\,, \tag{3}$$

$$\dot{V}_r = \frac{\Delta_r}{3} + \frac{(2-\beta)a^2}{2m^2k^2\beta}\left(\rho_m\Delta_m + \frac{4\rho_r}{3}\Delta_r\right) + \frac{4\rho_r a^2}{3m^2k^2\beta}\delta_{\mathcal{C}} + \frac{4(2-\beta)\rho_r a^2}{m^2k^2\beta(2+\beta)}HV_{\mathcal{C}}\,, \tag{4}$$

$$\ddot{\delta}_{\mathcal{C}} + \left(\frac{2\beta}{2+\beta} - 3\gamma\right)H\dot{\delta}_{\mathcal{C}} + \frac{k^2}{3a^2}(2+3\gamma)\delta_{\mathcal{C}} = -\frac{k^2(1+3\gamma)}{4\rho_r a^2}\left(\rho_m\Delta_m + \frac{4\rho_r}{3}\Delta_r\right), \tag{5}$$

$$\Delta_\lambda \equiv \delta_\lambda + 3HV_\lambda \qquad\qquad -\frac{k^2}{a^2}V_{\mathcal{C}} = \dot{\delta}_{\mathcal{C}}\,. \tag{6}$$

Gravitational potentials:

$$\Psi = -\frac{(2+\beta)a^2}{2m^2k^2\beta}\left(\rho_m\Delta_m + \frac{4\rho_r}{3}\Delta_r\right) - \frac{4\rho_r a^2}{3m^2k^2\beta}\left(\delta_{\mathcal{C}} + 3HV_{\mathcal{C}}\right),$$

$$\Psi - \Phi = -\frac{8\rho_r a^2}{3m^2k^2\beta}\left(\Delta_r + \frac{3\rho_m}{4\rho_r}\Delta_m + \delta_{\mathcal{C}} + \frac{6HV_{\mathcal{C}}}{2+\beta}\right).$$

Perturbations during matter domination

Matter ➡
$$\ddot{\Delta}_m + 2H\dot{\Delta}_m - \frac{\rho_m}{2m^2}\left(1+\frac{6\gamma}{\beta}\right)\Delta_m = \frac{4\rho_r(1+3\gamma)}{3m^2\beta}\,\delta_{\mathcal{C}}\,, \qquad (1)$$

Weyl fluid ➡
$$\ddot{\delta}_{\mathcal{C}} + \left(\frac{2\beta}{2+\beta} - 3\gamma\right)H\dot{\delta}_{\mathcal{C}} + \frac{k^2}{3a^2}(2+3\gamma)\delta_{\mathcal{C}} = -\frac{k^2(1+3\gamma)\rho_m}{4a^2\rho_r}\Delta_m\,, \qquad (2)$$

⬇ **5D**

If time derivatives of δ_C are assumed to be much smaller than its spatial gradients (on sub-Hubble scales) then

$$\delta_{\mathcal{C}} \approx -\frac{3\rho_m(1+3\gamma)}{4\rho_r(2+3\gamma)}\Delta_m\,, \qquad \Delta_m = \frac{\rho-\bar{\rho}}{\bar{\rho}}$$

$$-\frac{k^2}{a^2}\Psi \approx \frac{\rho_m\Delta_m}{2m^2}\left(1-\frac{1}{3\mu}\right)$$

$$-\frac{k^2}{a^2}\Phi \approx \frac{\rho_m\Delta_m}{2m^2}\left(1+\frac{1}{3\mu}\right)$$

which results in a closed equation for (1), namely

$$\boxed{\ddot{\Delta}_m + 2H\dot{\Delta}_m \approx \frac{\rho_m\Delta_m}{2m^2}\left(1+\frac{1}{3\mu}\right)}$$

Gravitational potentials

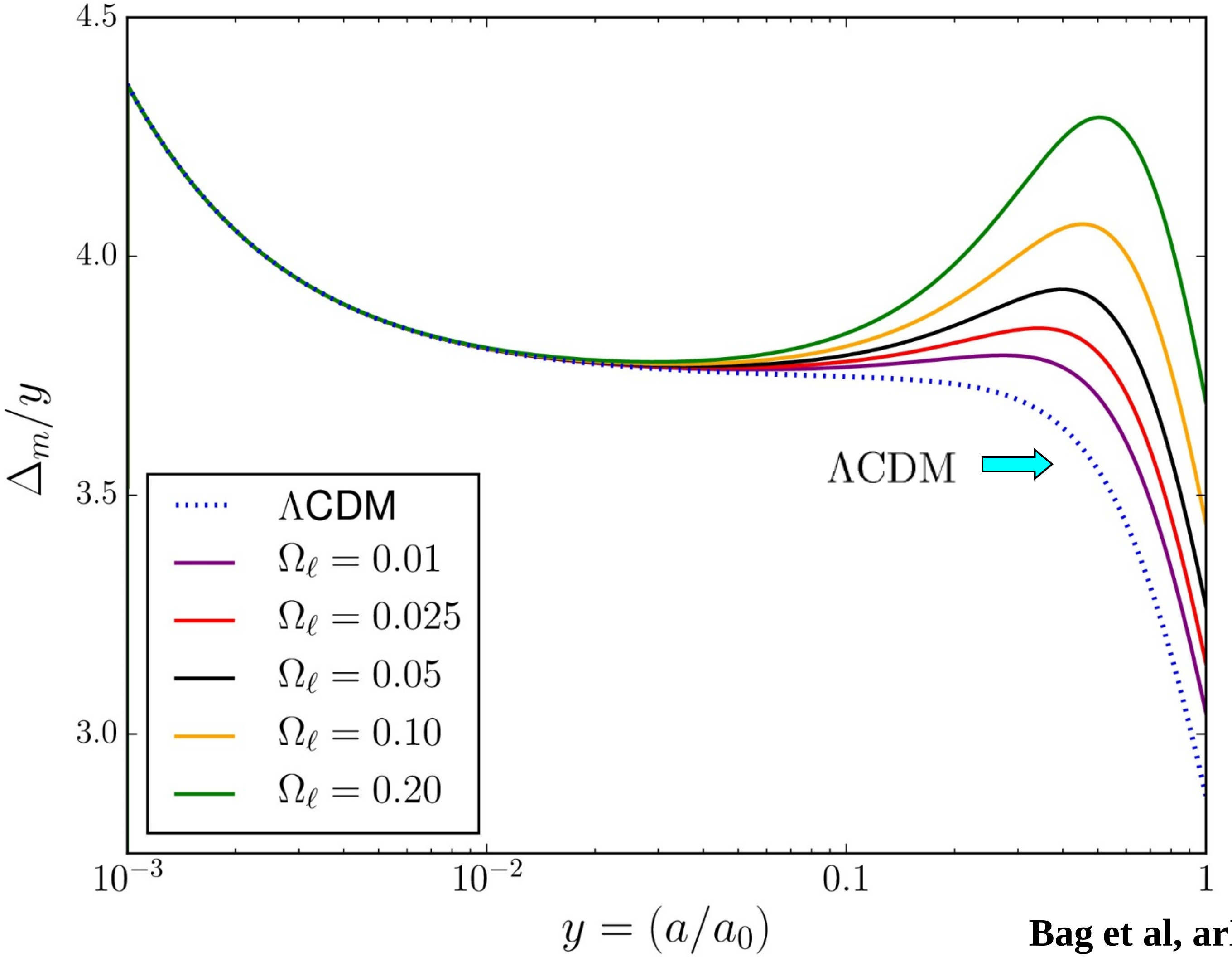
➡ **Quasi-static approximation**
[Koyama & Maartens, 2006]
(scale independent)

where $\mu \equiv 1+\ell H\left(1+\frac{\dot{H}}{3H^2}\right)$, $\ell = 2m^2/M^3$ describes the ratio of the four and five dimensional Planck mass

GR eqn ➡ $\ddot{\Delta}_m + 2H\dot{\Delta}_m \approx \frac{\rho_m\Delta_m}{2m^2}$

At late times, perturbations on the brane grow more rapidly than in ΛCDM.

Density perturbations on the brane are very sensitive to the value of the brane parameter Ω_ℓ, which depends on the ratio of the five- and four-dimensional gravitational couplings.



$$\Omega_\ell = \frac{1}{\ell^2 H_0^2}$$

$$\ell = 2m^2/M^3$$

$$\Omega_\ell \to 0$$

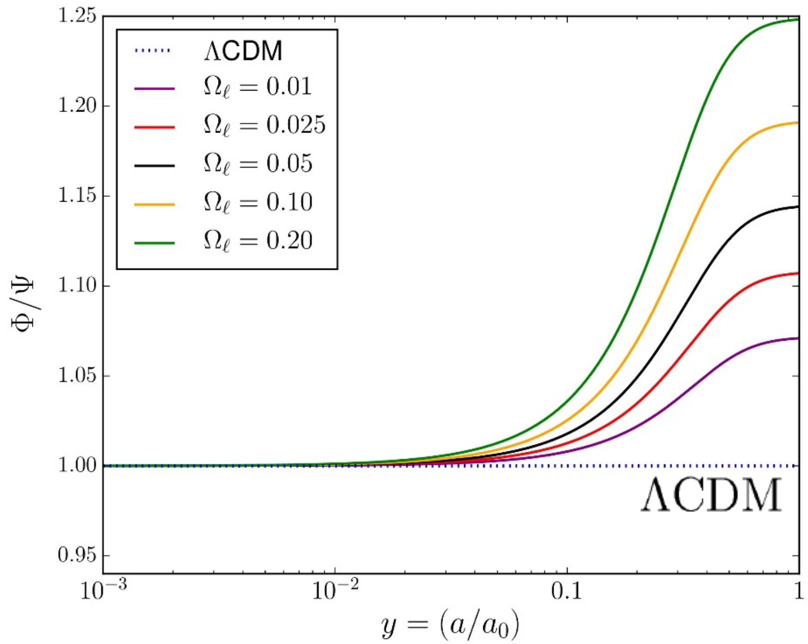
ΛCDM

Bag et al, arXiv:1603.01277

$$ds^2 = -(1+2\Phi)dt^2 + a^2(1-2\Psi)\gamma_{ij}dx^i dx^j$$

$\Psi = \Phi$ in ΛCDM, but $\Psi \neq \Phi$ on the brane.

Φ/Ψ increases with Ω_ℓ



Smoking gun test for Braneworld

Can be probed by <u>weak lensing</u>

See arXiv:1610.04606

Work in progress…..

$$S = M^3\left[\int_{\text{bulk}}(\mathcal{R} - 2\Lambda_{\text{b}}) - 2\int_{\text{brane}} K\right] + \int_{\text{brane}}(m^2R - 2\sigma) + \int_{\text{brane}} L\left(h_{ab}, \phi\right)$$

$$\Omega_\ell = \frac{1}{\ell^2 H_0^2}$$

$$\ell = 2m^2/M^3$$

M is the 5D Planck mass, and **m** is the 4D Planck mass.

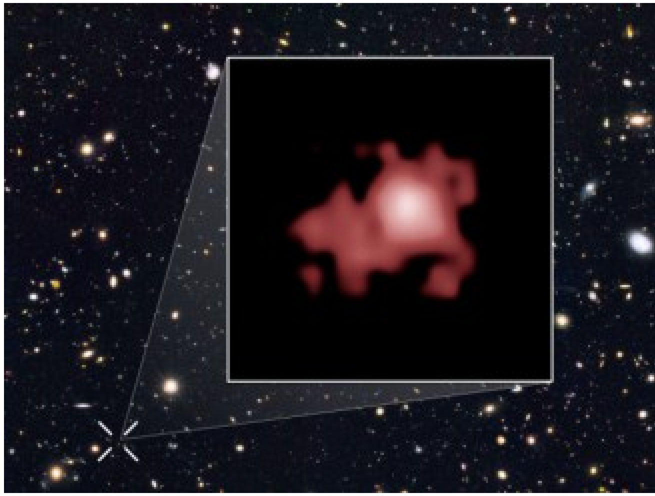
Bag, Viznyuk, Shtanov and VS, JCAP 07 (2016) 038

Potential challenges for the cosmological constant: Λ

1. An early population of galaxies and quasars at high redshift are being discovered.

GN-z11 a billion solar mass galaxy at z=11 has recently been discovered.
Its the furthest known galaxy in the Universe and also the **oldest**.
It formed when the universe was only 2% of its present age !!
(ie. 200-300 million years after the Big Bang).

Can such an old astrophysical object be accommodated in a ΛCDM Universe ?

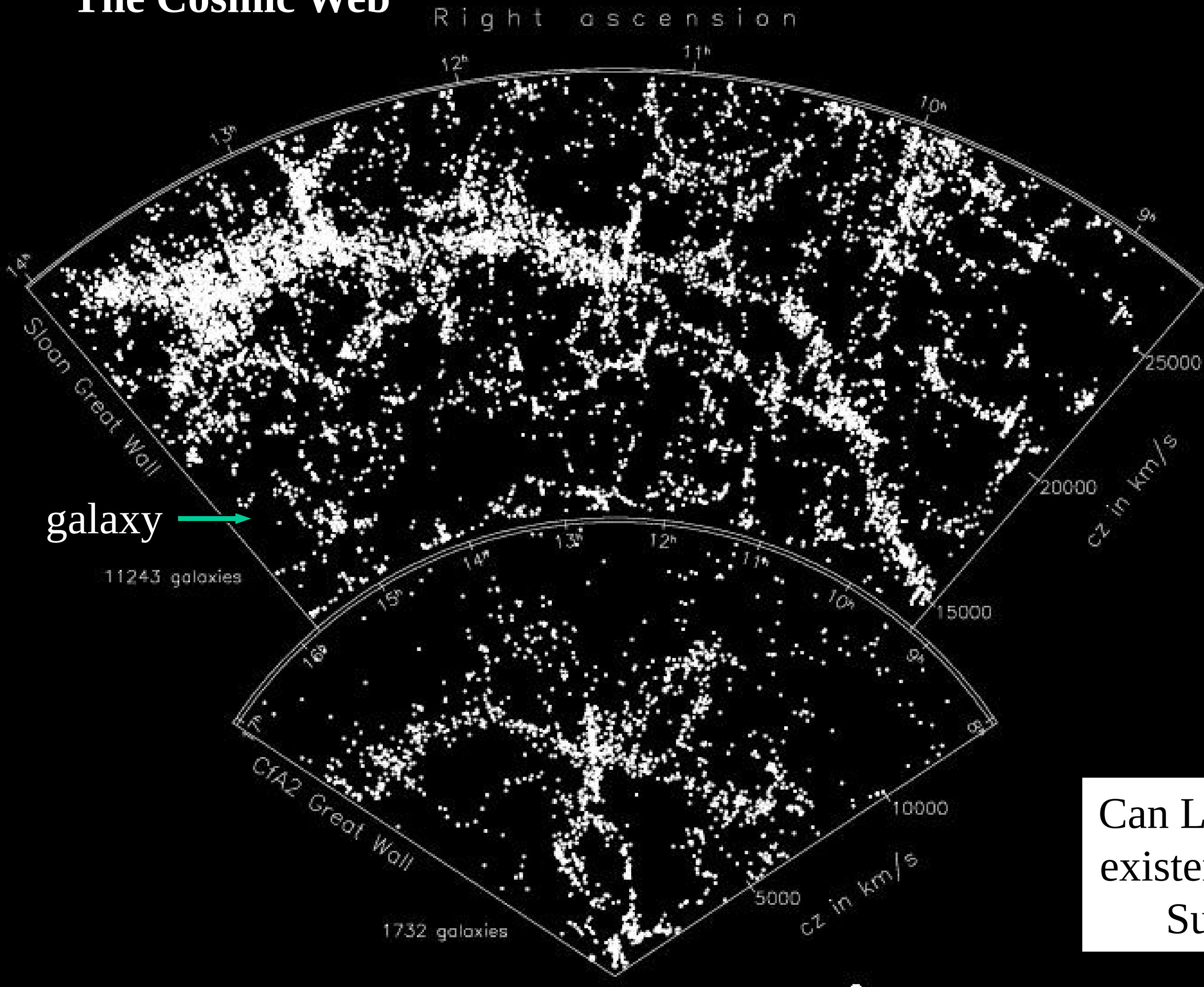


2. Over 50 quasars and ultra-luminous objects have been discovered at z > 6 including a quasar with mass $M \simeq 8 \times 10^8 \, M_\odot$ at z = 7.54

This indicates that massive black holes were already in place in the early Universe.

Can such old objects arise through gravitational instability in general relativity or are they easier to explain in a modified theory of gravity/cosmology ?

The Cosmic Web



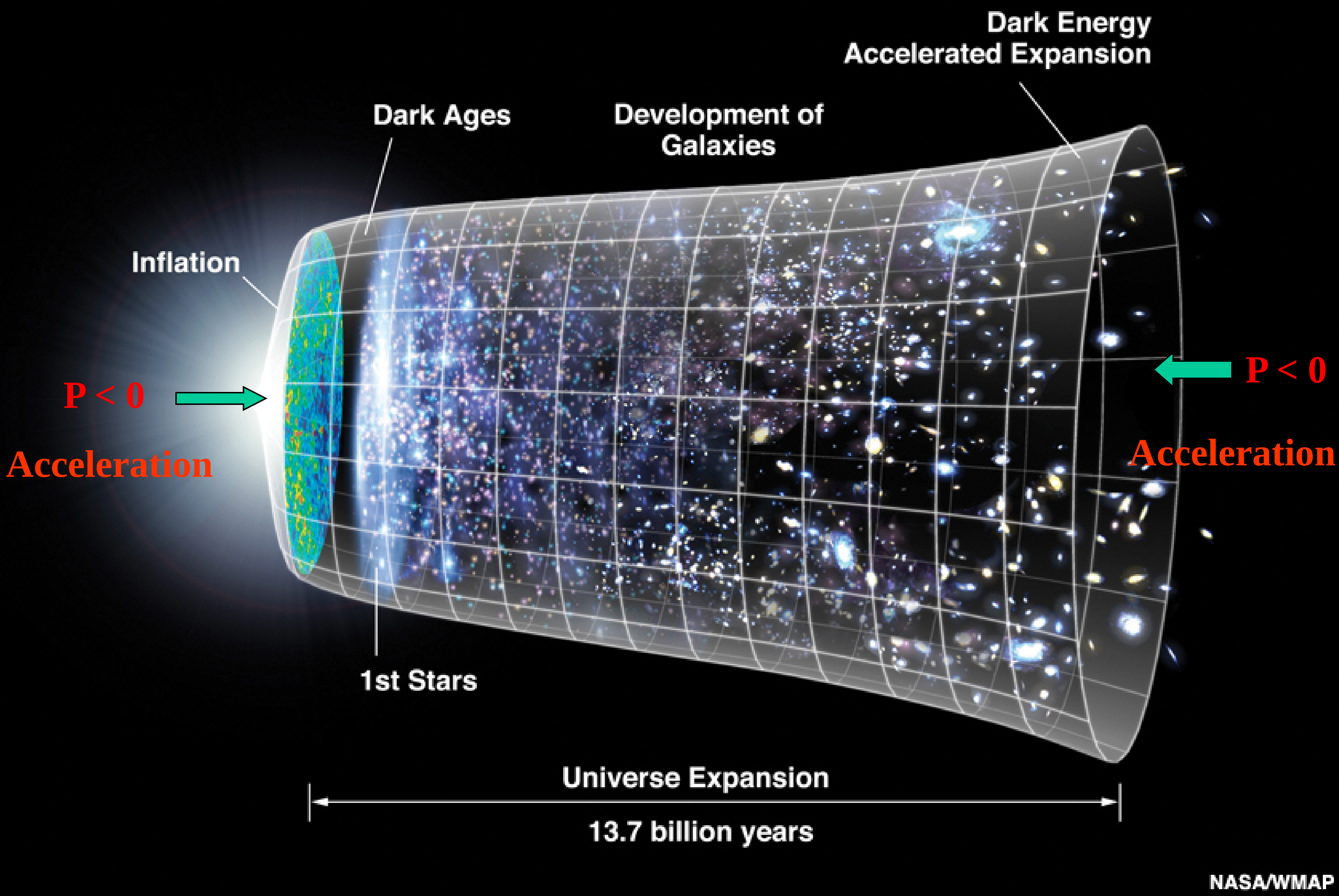
The Sloan Great Wall is the largest known structure in the Universe.

It is 1.37 billion light years distant and one billion Light years in size.

So it is several times larger than the CfA wall.

Can LCDM explain the existence of such large Superclusters ?

The Universe accelerates twice !



The beginning !

Inflation $\longrightarrow$ $p \simeq -\rho$!

Initial conditions (seeds) for structure formation

Dark matter

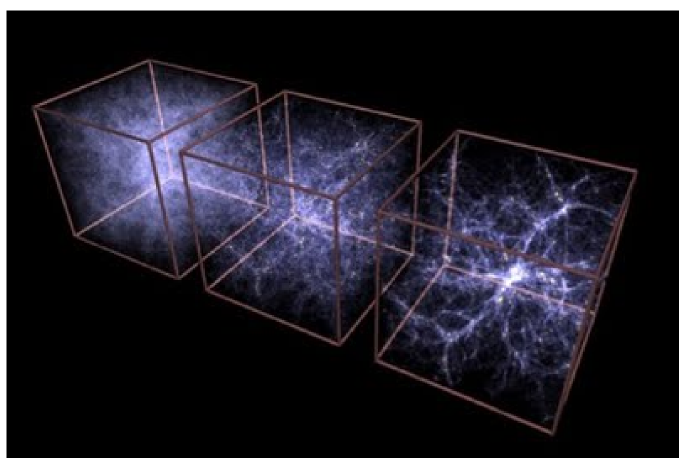
Dark energy

Speeds up structure formation

Cosmic Web

Slows down structure formation

Nature of dark matter: Hot, warm, cold affects cosmic web profoundly



The present !

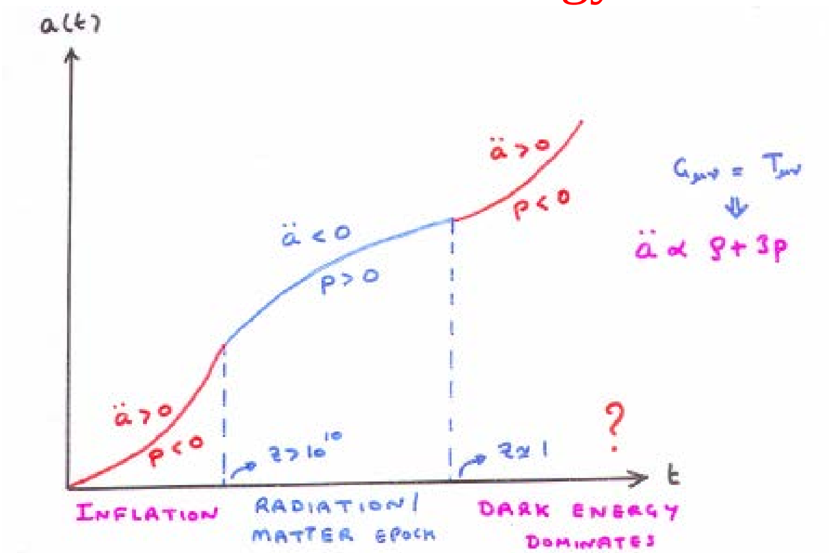
Open Questions circa 2017

- Is dark energy the cosmological constant or does dark energy evolve ?

- Are dark matter and dark energy related ?

Why cosmic coincidence ? $\Omega_m \sim 1/3, \; \Omega_{\rm DE} \sim 2/3$

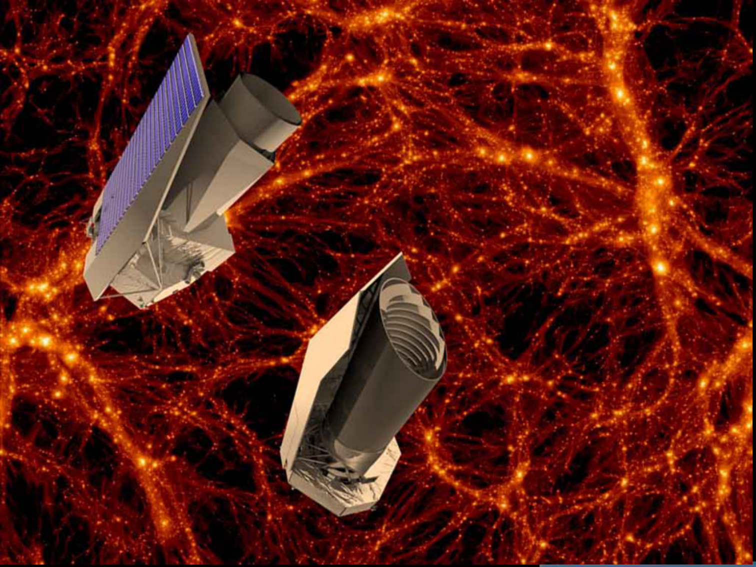
- Are inflation and dark energy related ? $p \simeq -\rho$ in both !



Why does the universe accelerate twice ?

- **Is cosmic acceleration due to modified gravity ?**

- Can LCDM account for the largest superclusters ? Early epoch of structure formation: BH with $M \sim 10^{10} M_\odot$ at $z = 6.3$ and 40 QSO's at z > 6 !



Big breakthroughs await us ….
100,000,000 galaxy redshifts soon !

Thousands of high redshift supernovae from DES and Euclid, and LSST.

Epoch of recombination from SKA !

Precise knowledge of Dark Energy Expected soon !

The significant problems we have cannot be solved at the same level of thinking with which we created them.

--- Albert Einstein

Perhaps this is also true for Dark Energy !

Thank You !!

Supernova Types

Type I

No H in spectra

Ia

Si Absorption line
@ 615nm

Found everywhere in the universe

Always same luminosity?

Ib

No Si

Ic

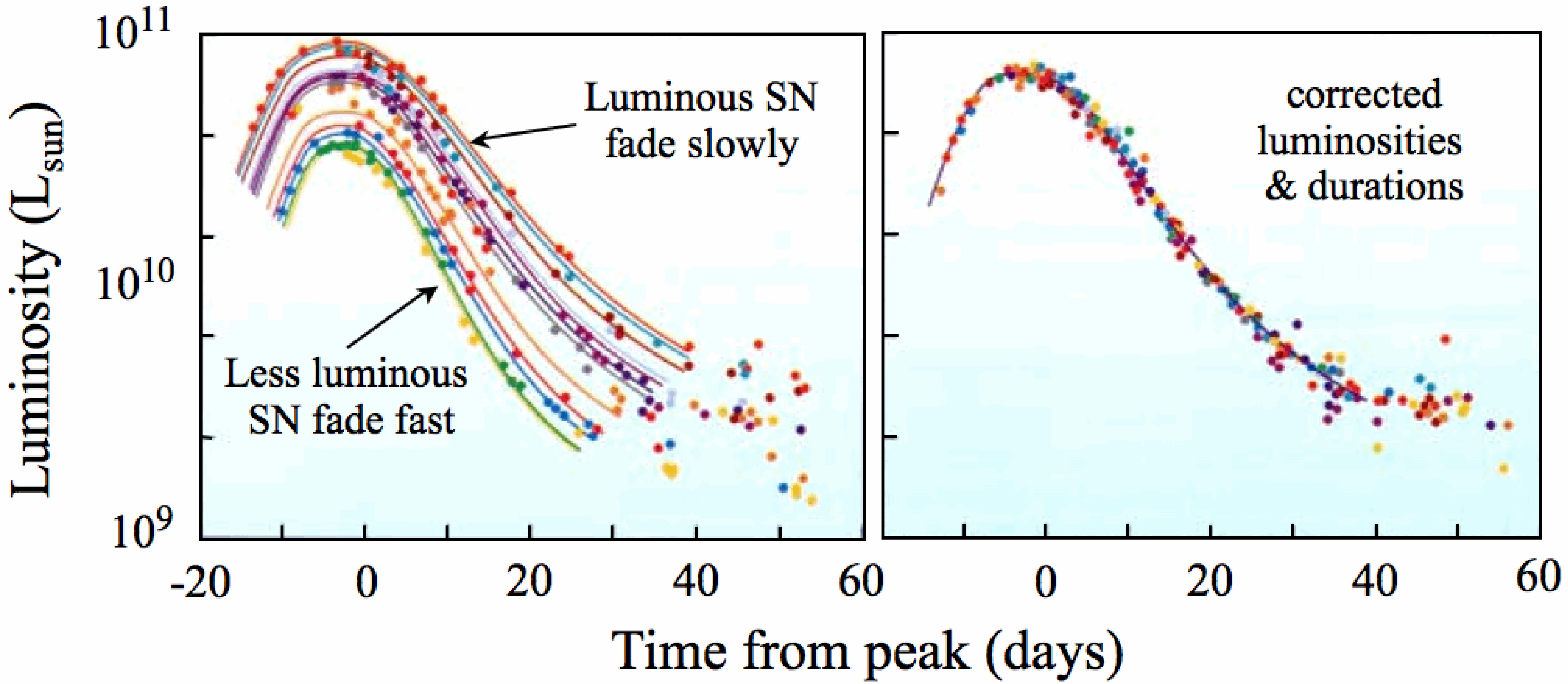
No Si,
No He

Found only in new star regions

Type II

H in spectra

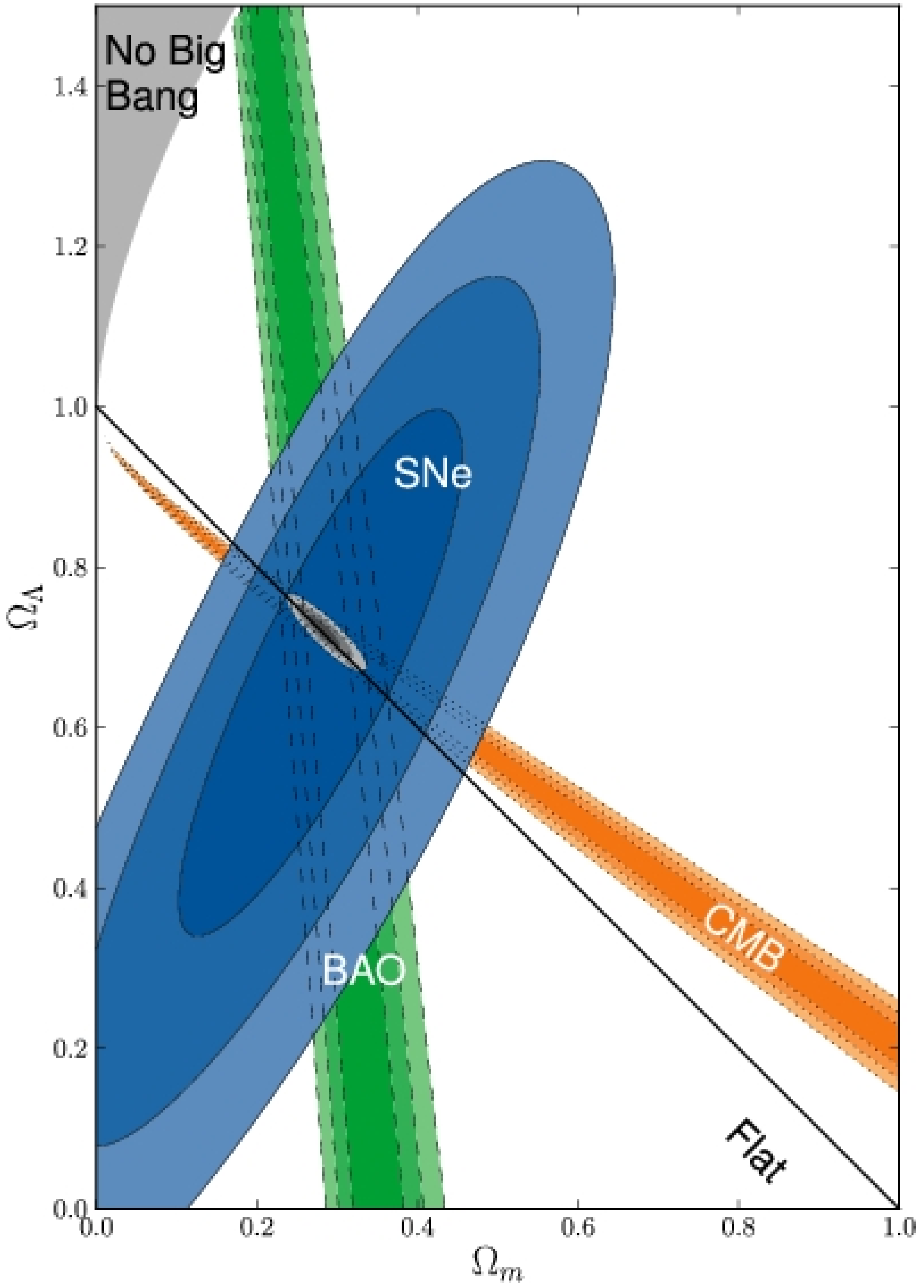
May be further subdivided based on light curves



SNIa luminosity caused by radioactive decay of Cobalt and Nickel into Iron.

The Cosmological Constant provides excellent agreement with observations !

$$\Omega_\Lambda \simeq 2/3,\ \Omega_m \simeq 1/3$$



Suzuki et al arXiv:1105.3470

New models of Dark Matter and Dark Energy from

$$\alpha - \mathrm{attractors}$$

Mishra, Sahni & Shtanov, JCAP 2017

The Lagrangian below is invariant under the O(1,1) group of transformations in the (χ, ϕ) space and also under the group of local conformal transformations.

$$\mathcal{L} = \sqrt{-g}\left[\frac{1}{2}\partial_\mu\chi\partial^\mu\chi + \frac{\chi^2}{12}R(g) - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{\phi^2}{12}R(g) - \frac{\tilde{\lambda}}{4}\left(\phi^2 - \chi^2\right)^2\right]$$

Fixing the local conformal gauge to $\chi^2 - \phi^2 = 6m_p^2$

the Lagrangian can be parametrized by

$$\chi = \sqrt{6}m_p \cosh\frac{\varphi}{\sqrt{6}m_p}\ , \quad \phi = \sqrt{6}m_p \sinh\frac{\varphi}{\sqrt{6}m_p}\ ,$$

and reduces to

$$\mathcal{L} = \sqrt{-g}\left[\frac{m_p^2}{2}R - \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \Lambda m_p^2\right]$$

which describes GR with the cosmological constant $\Lambda = 9\tilde{\lambda}m_p^2$

A conformally invariant generalization of the above Lagrangian can can be made

by the map $\Lambda \rightarrow F(\phi/\chi).m_p^4$

Kallosh and Linde, JCAP 07(2013)002

$$\mathcal{L} = \sqrt{-g}\left[\frac{m_p^2}{2}R - \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \Lambda m_p^2\right] \qquad \Lambda \to F(\phi/\chi).m_p^4$$

Which leads to $$\mathcal{L} = \sqrt{-g}\left[\frac{m_p^2}{2}R - \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - V(\varphi)\right]$$

where $$V(\varphi) = m_p^4 F\left(\tanh\frac{\varphi}{\sqrt{6}m_p}\right)$$

Kallosh, Linde and Roest introduced the $\alpha - \mathrm{attractor}$ family of potentials

following the prescription $V(\varphi) \to V\left(\varphi/\sqrt{\alpha}\right)$

so that $$V(\varphi) = m_p^4 F\left(\tanh\frac{\varphi}{\sqrt{6\alpha}m_p}\right)$$

$\mathrm{The}\ \alpha\ \mathrm{attractors}$ give rise to an interesting class of inflationary models which include Starobinsky inflation.

Kallosh, Linde & Roest, JHEP 11, 198 (2013)

The parameter α can be related to the curvature of the superconformal Kahler metric.

The class of potentials $$V(\varphi) = m_p^4 F\left(\tanh \frac{\varphi}{\sqrt{6\alpha} m_p}\right) \qquad (1)$$

can also give rise to very interesting new models for dark energy and dark matter.

Dark Energy models include:

1. The power-law tracker model with $V(\phi) = V_0 \coth \dfrac{\lambda\phi}{m_p}$

2. The oscillatory tracker model with $V(\phi) = V_0 \cosh \dfrac{\lambda\phi}{m_p}$

3. Exponential tracker model with

$$V(\phi) = V_0 \left[1 + \exp\left(-\frac{\lambda\phi}{m_p}\right)\right] \equiv 2V_0 \left[1 + \tanh \frac{\lambda\phi}{2m_p}\right]^{-1}$$

The tracker parameter λ is related to α in (1) by $\lambda = \sqrt{\dfrac{1}{6\alpha}}$.

1. The power-law tracker model with $V(\phi) = V_0 \coth^p \lambda\phi$

For small values $0 < \lambda\phi \ll 1$ one finds $V \simeq \dfrac{V_0}{(\lambda\phi)^p}$

Therefore at early times this model will behave like the inverse power law model

$V \propto \phi^{-p},\ p > 0$ suggested by Ratra and Peebles (1988).

Accordingly the equation of state (EOS) at early times is $w_\phi = \dfrac{pw_B - 2}{p + 2}$

w_B is the background equation of state: $w_B = 1/3$ (radiation), $w_B = 0$ (matter)

This tracker-like behaviour allows the present value $w_0 < -1/3$ to be reached from a large class of initial conditions.

However the original IPL model with $V \propto \phi^{-p}$

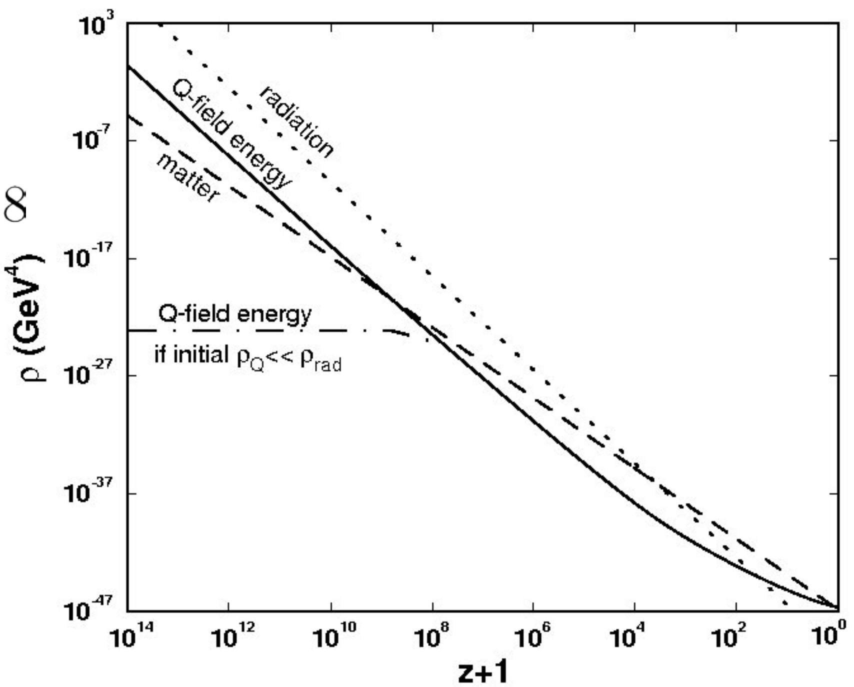
faces problems since the current value $w_0 < -0.8$

can only be reached if $p < 1$ which shrinks

the basin of attraction at high z.

This problem can be circumvented if

$$V(\phi) = V_0 \coth^p \lambda\phi$$



$$\{w, w'\}$$

$$w' = \frac{dw}{d\log a} \equiv \frac{\dot{w}}{H}$$

Oscillatory tracker: $V(\phi) = V_0 \cosh \lambda\phi$

For large values $\lambda\phi \gg 1$ (corresponding to early times) $V \sim V_0\, e^{\lambda\phi}$

This potential has the interesting property that for $\lambda^2 > 3(1+w_{\rm B}),\ w_\phi = w_{\rm B}$

i.e. **dark energy mimics the background equation of state !**

The corresponding fractional density satisfies $$\frac{\rho_\phi}{\rho_{\rm B} + \rho_\phi} = \frac{3(1+w_{\rm B})}{\lambda^2}$$

where cosmological nucleosynthesis constraints impose the lower bound $\lambda \geq 5$.

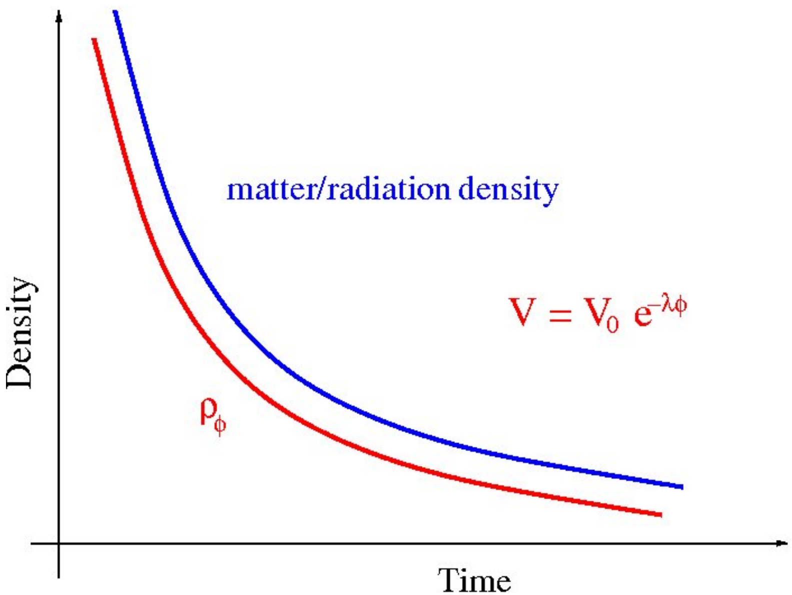
Unfortunately the exponential potential

$$V \sim V_0\, e^{\lambda\phi}$$

cannot account for late-time acceleration

since $w_0 < 0$ is **never reached** !

This is easily corrected if $V \propto \cosh \lambda\phi$



For small values $\lambda|\phi| \ll 1$ the potential $V(\phi) = V_0 \cosh \lambda\phi$ has the interesting form

$$V(\phi) \simeq V_0 \left[1 + \frac{(\lambda\phi)^2}{2}\right]$$

This suggests that $V(\phi) \simeq V_0$, when $\lambda\phi \to 0$.

In other words, dark energy behaves like a **cosmological constant** at late times.

The presence of the ϕ^2 term ensures that the approach to $w_\phi = -1$ is **oscillatory**.

Braneworld models of Dark Energy

$$S = M^3 \left[\int_{\rm bulk} (\mathcal{R} - 2\Lambda_{\rm b}) - 2 \int_{\rm brane} K \right] + \int_{\rm brane} \left(m^2 R - 2\sigma \right) + \int_{\rm brane} L\left(h_{ab}, \phi\right)$$

5D (bulk)

4D (brane)

M is the 5D Planck mass, and **m** is the 4D Planck mass.

σ is the brane tension

$\ell = 2m^2/M^3$ is a new length scale !

$\Lambda_{\rm b}$ is the bulk cosmological constant.

K is the trace of the extrinsic curvature tensor on the brane,

The above action includes several important cosmologies as *special cases*:

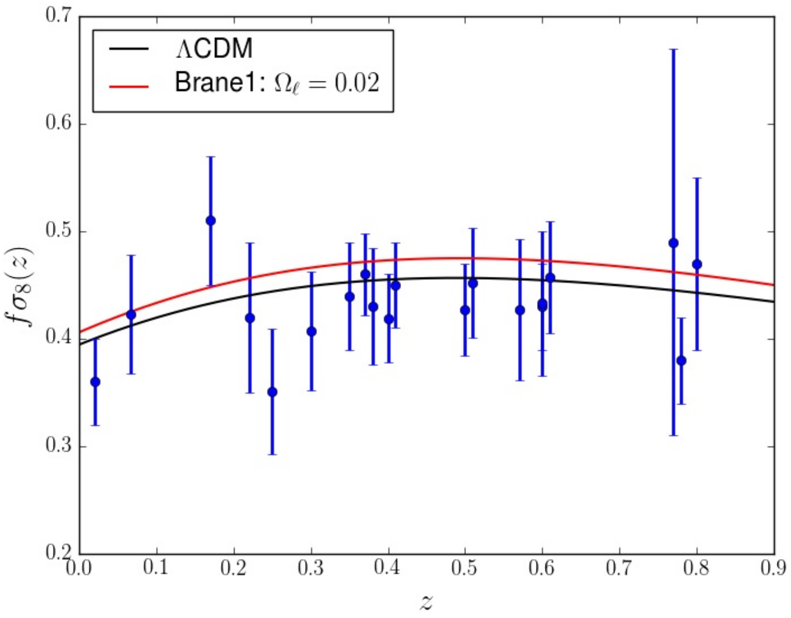
1. **General Relativity**: $M = 0, \Lambda_{\rm b} = 0$

2. **DGP brane**: $\Lambda_{\rm b} = 0, \sigma = 0$

3. **Randall-Sundrum** model: $m = 0 \Rightarrow \ell = 0$

Field eqn. in an FRW Universe:

$$m^4 \left(H^2 + \frac{\kappa}{a^2} - \frac{\rho + \sigma}{3m^2} \right)^2 = M^6 \left(H^2 + \frac{\kappa}{a^2} - \frac{\Lambda_{\rm b}}{6} - \frac{C}{a^4} \right)$$

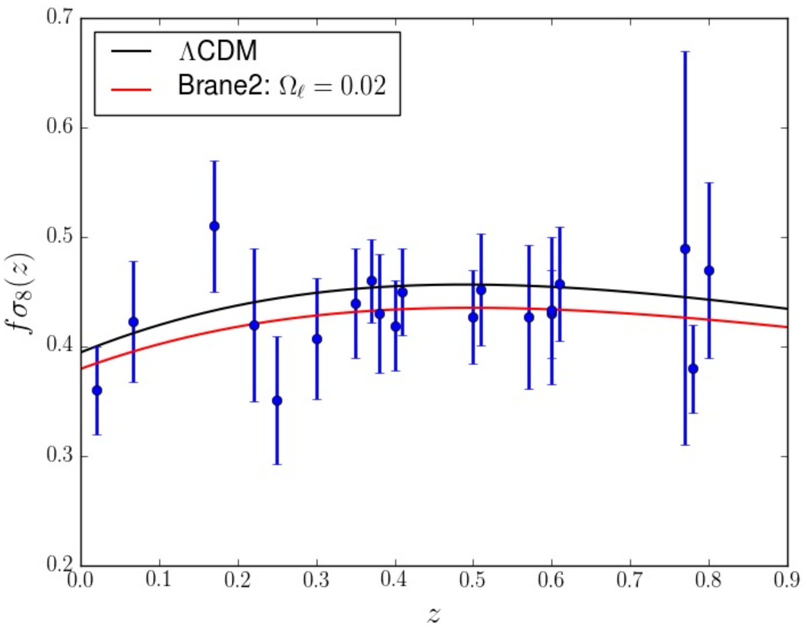


Different embeddings of the brane in the bulk lead to different EOS's for dark energy.

$$w_0 < -1$$

Comparison with observations:
work in progress…..

Alam, Bag, Sahni, Shtanov, Viznyuk (2017)

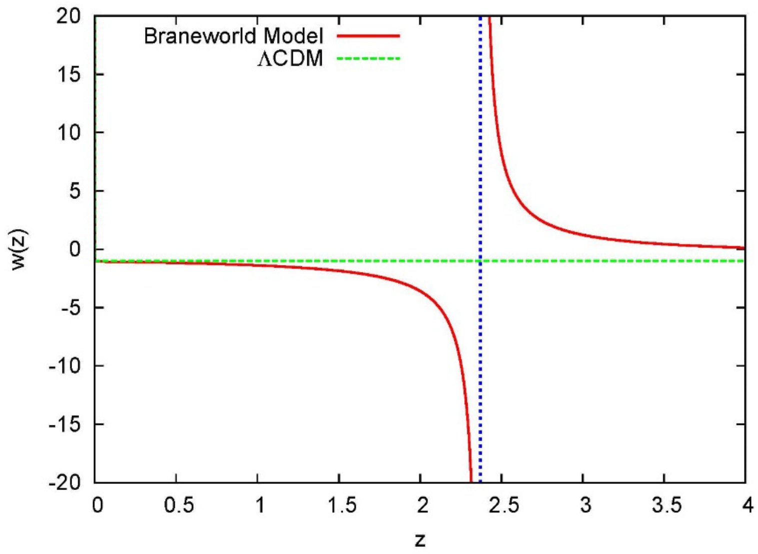


$$w_0 > -1$$

• The cosmological constant diminishes at high z in screened DE models.

$$H^2(z) = \underbrace{\frac{\Lambda}{3} - f(z)}_{\rho_{\rm DE}} + \kappa\rho_{0m}(1+z)^3, \quad f(z) > 0$$

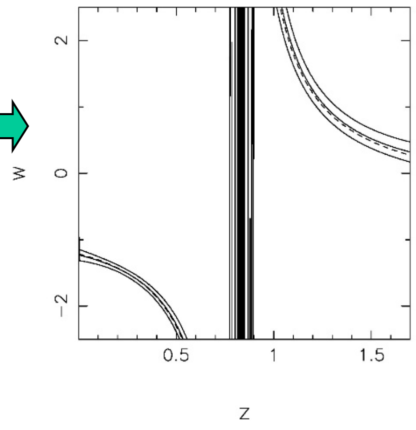
$w \equiv p_{DE}/\rho_{DE}$ will have a pole when $\rho_{\rm DE}(z_p) = 0$ and $w(z_p) \rightarrow \infty$!



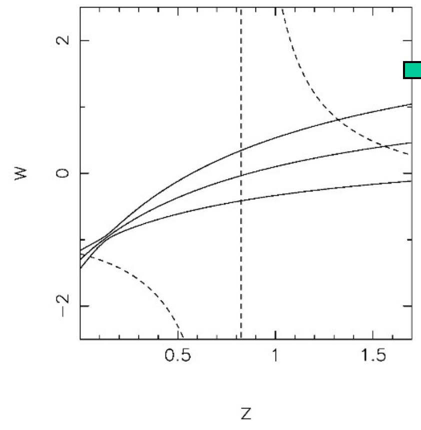
Smoking gun test of screened models.

Can be established by applying non-parametric reconstruction to future (Euclid) data.

Non-parametric ansatz works well



$$w(z) = w_0 + (1-a)w_1$$



CPL cannot `see' Pole.

$$D_L^s(z) = \int D_L(z') \exp\left(-\frac{|z-z'|}{2\Delta^2}\right) dz'$$

Shafieloo et al MNRAS (2006)

Observational tests of Dark Energy usually rely on an accurate measurement of either the angular size distance or the luminosity distance:

$$\mathcal{F} = \frac{L}{4\pi D_L^2}\ , \quad D_L(z) = (1+z)\int_0^z \frac{dz'}{H(z')}\ .$$

One can then reconstruct the Hubble parameter through

$$H(z) = \left[\frac{d}{dz}\left(\frac{D_L(z)}{1+z}\right)\right]^{-1}\ .$$

Or by determining H(z) directly from Baryon acoustic oscillations (BAO).

Differentiating H(z) we can reconstruct the equation of state of DE

$$w \equiv p_{DE}/\rho_{DE} = \frac{2q(x)-1}{3(1-\Omega_m(x))} \equiv \frac{\frac{2x}{3}\frac{d\log H}{dx} - 1}{1-(H_0/H)^2\Omega_{0m}x^3}\ ,\ x = 1+z\ .$$

BUT $w(z)$ **will be a noisier quantity than** $H(z)$ **since a differentiation is needed for the reconstruction** $H(z) \to w(z)$.

Also note that $H(z)$ is independent of the value of Ω_{0m} while $w(z)$ is not !

Therefore uncertainties in Ω_{0m} affect the reconstruction of $w(z)$ much more significantly than the reconstruction of $H(z)$.

In practice observational quantities such as $D_L(z_i)$ are noisy and known only at discrete values of the redshift. Thus it is impossible to directly differentiate them. Therefore, to convert from $D_L(z_i)$ to $H(z)$ one requires some sort of smoothing procedure.

This is usually accomplished using either parametric or non-parametric methods.

$\boxed{\text{Parametric reconstruction}}$ [A] Fitting functions to $D_L(z)$.

1. The simplest Taylor series: $\frac{D_L(z)}{1+z} = \sum_{i=1}^{N} a_i z^i$, does not work since to accurately determine $H(z), w(z)$ one must make N large which increases the errors of reconstruction [Huterrer and Turner, PRD 1999]. Better convergence is achieved by

$$D_L = \frac{c}{H_0}\left[y + Ay^2 + By^3 + \cdots\right] , \ y = \frac{z}{1+z} ,$$

[Cattoen & Visser CQG 2007,2008; Guimaraes & Lima, 2010]

2. A versatile 2 parameter ansatz is

$$\frac{H_0 D_L(z)}{1+z} = 2\left[\frac{x - A_1\sqrt{x} - 1 + A_1}{A_2 x + A_3\sqrt{x} + 2 - A_1 - A_2 - A_3}\right] , \quad x = 1+z ,$$

which exactly reproduces both CDM $(\Omega_m = 1)$ and the steady-state model $(\Omega_\Lambda = 1)$.

[Saini *et al* PRL 85, 1162, 2000]

B. Fitting function to the dark energy density:

$$\rho_{\rm DE} = A_1 + A_2 x + A_3 x^2 \ , \ \ x = 1 + z \ .$$

This leads to the following ansatz for H(z): [VS *et al* 2003, Barboza & Alcaniz, 2011]

$$H(x) = H_0 \left[\Omega_m x^3 + A_1 + A_2 x + A_3 x^2\right]^{1/2} \ .$$

C. Fitting functions to the equation of state. The simple Taylor expansion $w(z) = \sum_{i=1}^{N} w_i z^i$, with N=1 fares much better than the Taylor expansion for $D_L(z)$.

But $w(z) = w_0 + w_1 z$ is of limited utility since its only valid for $z \ll 1$.

A much more versatile ansatz is $w(a) = w_0 + w_1(1-a) = w_0 + w_1 \frac{z}{1+z}$ → **CPL ansatz**

where the parameters $w_0, \ w_1$ are obtained after substituting into:

$$H^2(z) = H_0^2[\Omega_m(1+z)^3 + \Omega_{\rm DE}]^2 \ , \ \Omega_{\rm DE} = (1-\Omega_m)\exp\left\{3\int_0^{x-1} \frac{1+w(z,a_i)}{1+z} dz\right\} .$$

$$D_L(z) = (1+z)\int_0^z \frac{dz'}{H(z')} \ .$$

[Chevalier & Polarski 2001; Linder 2003]

D. Fitting functions to the deceleration parameter q(z) have been discussed in:
Ishida, Reis, Toribio & Waga, Astropart. Phys., 2008 (and references therein).

E. Powerful Non-parametric methods: **see review Sahni & Starobinsky (2006).**

For Quintessence, one can reconstruct the potential $V(\phi)$ from observations of $H(z)$.

$$H^2 = \frac{8\pi G}{3}\left[\rho_m + \frac{1}{2}\dot{\phi}^2 + V(\phi)\right] , \quad \dot{H} = -4\pi G(\rho_m + \dot{\phi}^2)$$

which can be rewritten as

$$\frac{8\pi G}{3H_0^2}V(x) = \frac{H^2}{H_0^2} - \frac{x}{6H_0^2}\frac{dH^2}{dx} - \frac{1}{2}\Omega_{0\mathrm{m}}\, x^3,$$

$$\frac{8\pi G}{3H_0^2}\left(\frac{d\phi}{dx}\right)^2 = \frac{2}{3H_0^2 x}\frac{d\ln H}{dx} - \frac{\Omega_{0\mathrm{m}}x}{H^2}, \quad x \equiv 1+z \ .$$

Integrating, we determine $\phi(z)$. Inverting $\phi(z) \rightarrow z(\phi)$ and substituting into $V(x)$ allows us to reconstruct $V(\phi)$ from $H(z)$.

However, this reconstruction is valid only when $H^2(z) > H_0^2\left[1 + \Omega_{0m}(1+z)^3\right]$,

which is a restatement of the weak energy condition: $\rho_\phi + p_\phi \geq 0$,

which is satisfied by the scalar field.

[Saini et al, PRL 2000, VS & Starobinsky, IJMP 2006]

However, Cosmological Reconstruction is NOT UNIQUE !

The same expansion history, $H(z)$, may result from two very different dark energy models !

Example 1. DE with a constant equation of state $-1 < w < 0$ is described by the potential:

$$V(\phi) = \frac{3H_0^2(1-w)(1-\Omega_{m0})^{1/|w|}}{16\pi G\Omega_{m0}^{\alpha}} \sinh^{-2\alpha}\left(|w|\sqrt{\frac{6\pi G}{1+w}}(\phi - \phi_0 + \phi_1)\right) ,$$

where

$$\alpha = \frac{1+w}{|w|}, \quad \phi_0 = \phi(t_0), \quad \phi_1 = \sqrt{\frac{1+w}{6\pi G}}\frac{1}{|w|}\ln\frac{1+\sqrt{1-\Omega_{m0}}}{\sqrt{\Omega_{m0}}} .$$

Consequently, a universe filled with such a scalar field will have properties which are **identical** to those of a different universe filled with a tangled network of cosmic strings $(w = -1/3)$ or domain walls $(w = -2/3)$.

Example 2. The Chaplygin gas which has $p = -A/\rho$ can be described by a minimally coupled scalar field with the potential

$$V(\phi) = \frac{\sqrt{A}}{2}\left(\cosh(2\sqrt{6\pi G}\phi) + \frac{1}{\cosh(2\sqrt{6\pi G}\phi)}\right) ,$$

and associated with the Lagrangian density $\mathcal{L} = \frac{1}{2}\dot{\phi}^2 - V(\phi)$.

[Kamenschik, Moschella and Pasquier, PLB, 2001, Gorini et al, PRD 2005]

However the Chaplygin gas can also be modeled completely differently using a scalar field with the Born-Infeld kinetic term:

$$\mathcal{L} = -V_0\sqrt{1 - \phi_{,\mu}\phi^{,\mu}} .$$

[Bilic et al, PLB 2002, Frolov et al, PLB 2002]

This once more illustrates the fact that the equation of state $w(z)$ does not uniquely define an underlying field-theoretic model !

One of the most EXCITING observational discoveries of the past decade is that the

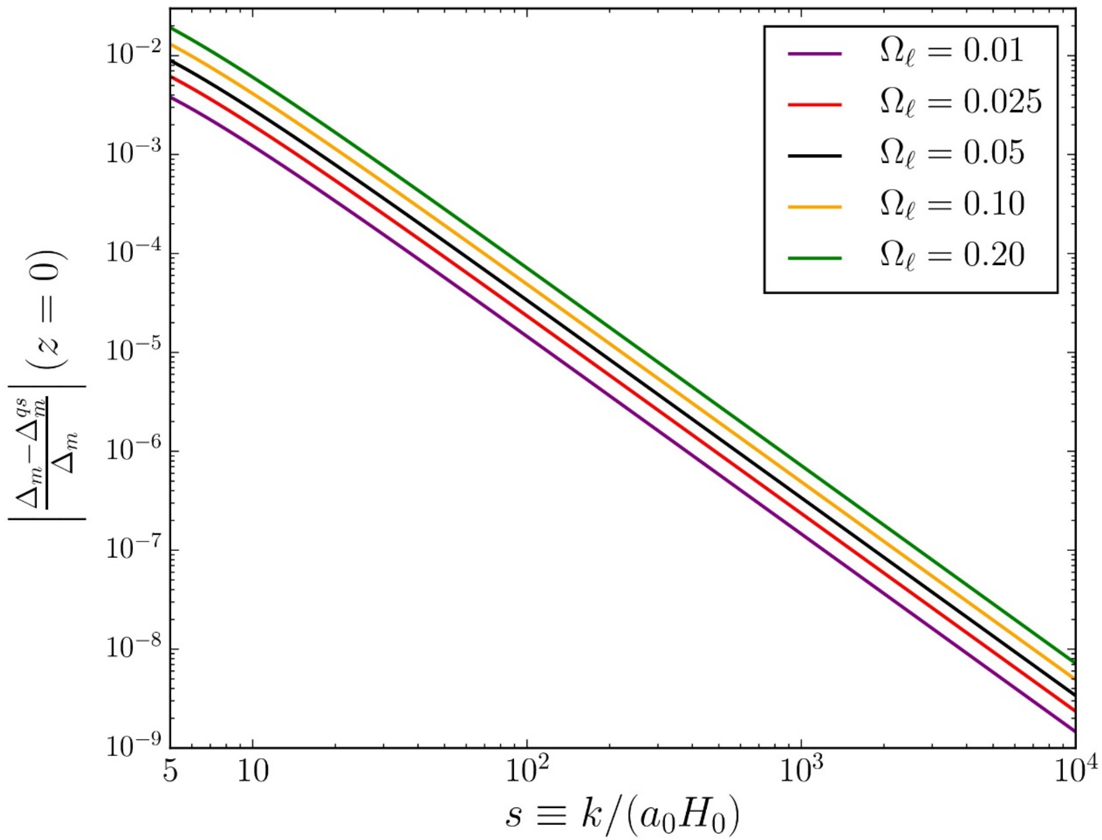
Universe is Accelerating !

The source responsible for Cosmic Acceleration is presently unknown is called

Dark Energy

Dark Energy has large negative pressure and could account for up to 70% of the total matter density in the Universe !!

Quasi-static approximation is very accurate !



Bag et al. arXiv:

The fractional difference between perturbations on the brane obtained by solving the exact system of equations (Δ_m) and by using the quasi-static approximation ($\Delta_m^{\rm qs}$) is shown at the present epoch ($z = 0$) for different values of $\Omega_\ell = 0.025$ and $s = k/a_0 H_0$. The accuracy of the quasi-static approximation increases for higher values of s and lower values of Ω_ℓ. (The limit $\Omega_\ell \to 0$ corresponds to ΛCDM.

$$H^2 + \frac{\kappa}{a^2} = \frac{\rho+\sigma}{3m^2} + \underline{\frac{2}{\ell^2}\left[1 \pm \sqrt{1 + \ell^2\left(\frac{\rho+\sigma}{3m^2} - \frac{\Lambda_b}{6} - \frac{C}{a^4}\right)}\,\right]}$$

$\ell = 2m^2/M^3$ is a new length scale ! $m^2 = \frac{1}{8\pi G}$

The underlined terms make Braneworld expansion different from LCDM.

The $\pm$ sign represents two branches of solutions.

+ sign corresponds to DGP cosmology, while `-' sign represents the normal branch.
Normal branch shows phantom behaviour: $w_{\rm eff} < -1$

Setting $C = 0,\ \Lambda_{\rm b} = 0,$ we find $h^2 = \Omega_{0m}(1+z)^3 + \underbrace{\Omega_\Lambda - f(z)}_{\text{Screened DE}}$

where $\Omega_\Lambda = \Omega_\sigma + 2\Omega_\ell$ $\Longrightarrow$ effective cosmological constant

$$\Omega_\sigma = 1 - \Omega_{0m} + 2\sqrt{\Omega_\ell}$$

$$f(z) = -2\sqrt{\Omega_\ell}\sqrt{\Omega_{0m}(1+z)^3 + \Omega_\sigma + \Omega_\ell}$$

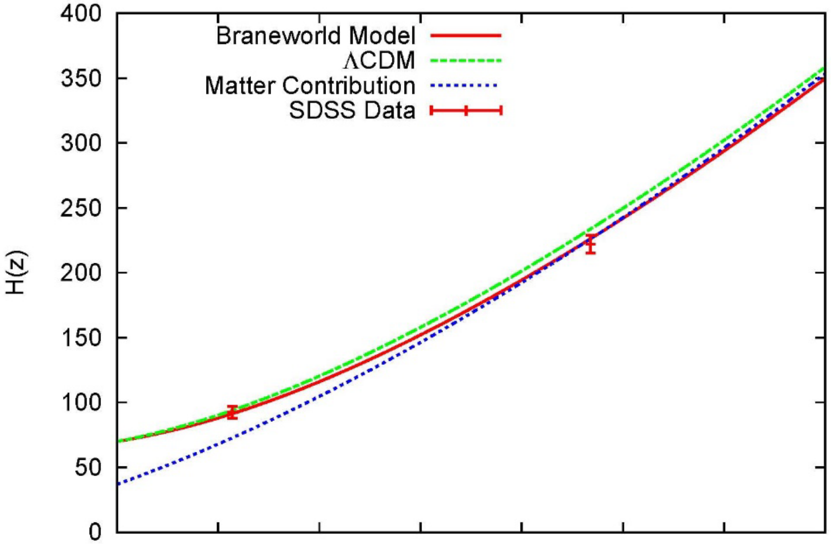
$\Omega_\sigma = \frac{\sigma}{3m^2 H_0^2}\,,\ \Omega_\ell = \frac{1}{\ell^2 H_0^2}$

Screening term, its value increases with redshift !

$$h^2 = \Omega_{0m}(1+z)^3 + \underbrace{\Omega_\Lambda - f(z)}_{\Omega_{\rm DE}}$$ (Screened DE)

This might explain why the SDSS value: $H(z=2.34) = 222 \pm 7\ \mathrm{km/sec/Mpc}$

is lower than the LCDM value $H(z=2.34) = 238\ \mathrm{km/sec/Mpc}$



New feature of the Phantom brane: Cosmological constant can be cancelled, leading to $\dfrac{h^2(z_*)}{(1+z_*)^3} = \Omega_{0m}$

$$\Rightarrow \frac{h^2(z_*)h_{100}^2}{(1+z_*)^3} = \Omega_{0m}h_{100}^2 \quad , h_{100} = \frac{H_0}{100}$$

RHS is determined from CMB:

$$\Omega_{0m}h_{100}^2 = 0.142 \pm 0.002$$

Substituting $H(z=2.34) = 222 \pm 7\ \mathrm{km/sec/Mpc}$ one gets

$\dfrac{h^2(z_*)h_{100}^2}{(1+z_*)^3} = 0.132 \pm 0.008$ which is slightly lower than the CMB value 0.142 ± 0.002

This suggests $\Omega_{\rm DE} \simeq 0$, at $z \simeq 2$.

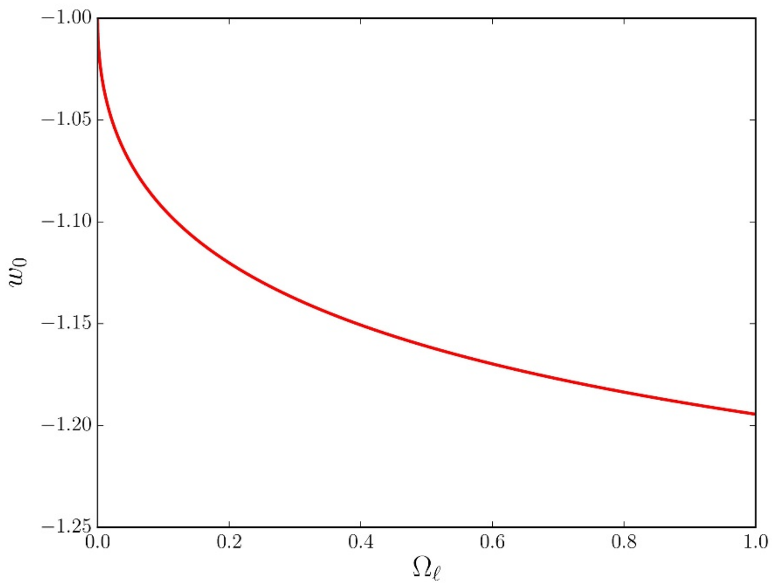
Equation of state of dark energy

$$H^2 \simeq \frac{8\pi G}{3}(\rho_m + \rho_{\rm DE}) \Rightarrow \rho_{\rm DE} \simeq \frac{3}{8\pi G}H^2 - \rho_m \Rightarrow \quad \rho_{\rm DE} = \frac{3H^2}{8\pi G}(1-\Omega_m)$$

$$\frac{\ddot a}{a} \simeq -\frac{4\pi G}{3}(\rho_m + \rho_{\rm DE} + 3P_{\rm DE}) \Rightarrow \quad p_{\rm DE} = \frac{H^2}{4\pi G}\left(q - \frac{1}{2}\right) \qquad q \equiv -\ddot a/aH^2$$

$$\boxed{w_{\rm eff}(z) \equiv \frac{P_{\rm DE}}{\rho_{\rm DE}} = \frac{2q(z)-1}{3\,(1-\Omega_m(z))}}$$

In the Phantom brane $\quad w_0 \equiv w_{\rm eff}(z=0) = -1 - \dfrac{\Omega_{0m}}{1-\Omega_{0m}}\left(\dfrac{\sqrt{\Omega_\ell}}{1+\sqrt{\Omega_\ell}}\right)$



$\Rightarrow \quad w_0 < -1$

w_0 decreases as Ω_ℓ increases, $\quad \Omega_\ell = \frac{1}{\ell^2 H_0^2}$

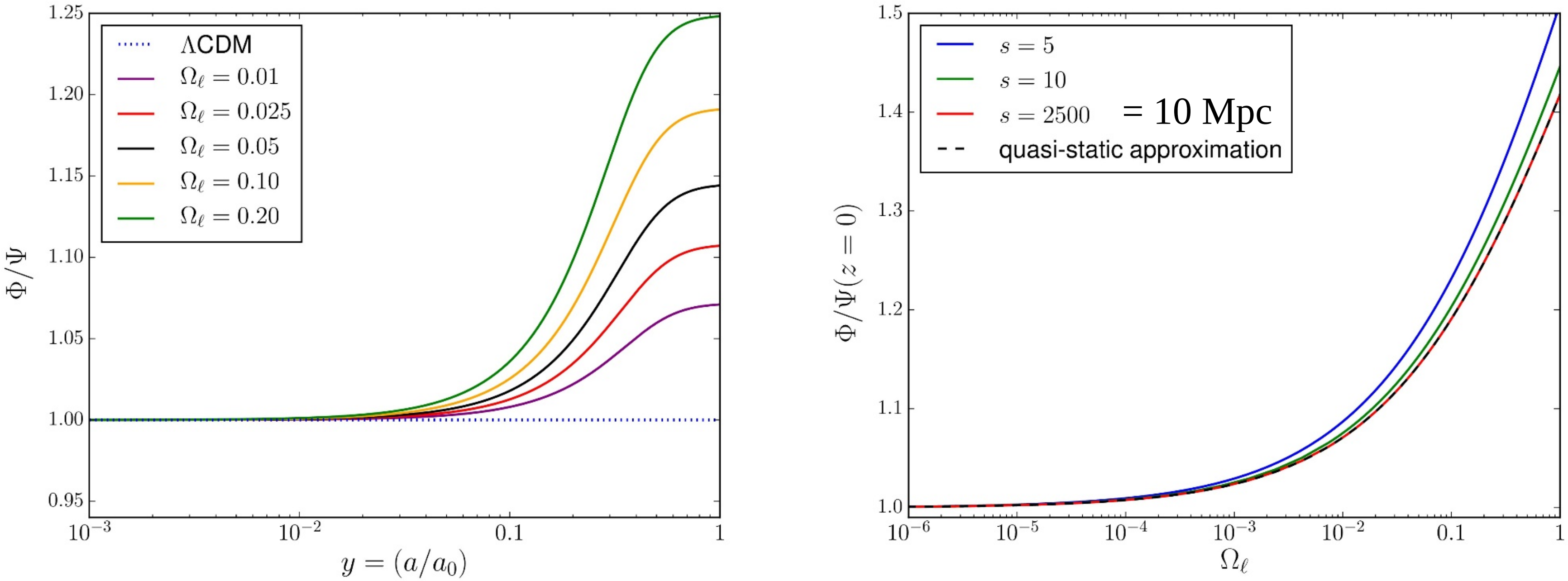
$$\ell = 2m^2/M^3$$

$\Omega_\ell = 0$ corresponds to ΛCDM

Alam, Bag & VS [arXiv:1605.04707]

Quasi-static approximation gives **sub-percent** accuracy for scales below 10 Mpc.

$$ds^2 = -(1+2\Phi)dt^2 + a^2(1-2\Psi)\gamma_{ij}dx^i dx^j$$



The accuracy of the quasi-static approximation increases for higher values of $s = k/a_0H_0$ and lower values of Ω_ℓ. (The limit $\Omega_\ell \to 0$ corresponds to ΛCDM.

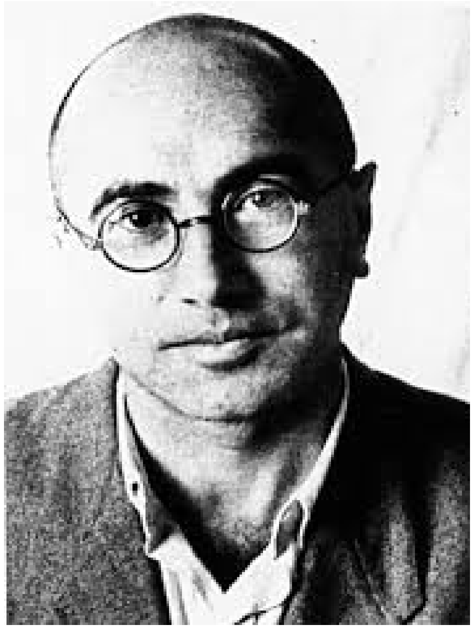
2016 is an interesting year since it lies between three centeneries:

- Zeldovich centenary (1914-1987)
- 100 years of General Relativity (1915)
- 100 years of Einstein’s cosmological constant (1917)

The Cosmological constant (Dark Energy) is a theme that is common to all three !

Yakov Borisovich Zeldovich was enormously talented !

Major contributions in:



- Chemical physics (adsorption & catalysis)
- Theory of shock waves
- Thermal explosions
- Theory of flame propogation
- Theory of combustion & detonation
- Nuclear physics
- Particle physics
- Astrophysics and Cosmology

(`Zeldovich number' in combustion theory)

Total scientific output includes over 500 research articles and 20 books.

Hawking: ``*Now I know you are a real person and not a group of scientists like Bourbaki"*

Remarkably, Zeldovich received no formal university education !

He graduated from high school at the age of 15 after which he joined the *Institute for Mechanical Processing of Useful Minerals* to train as a laboratory assistant.

The Soviet scientist Ioffe was very impressed by the young Zeldovich and wrote a letter to his institute requesting that Zeldovich be ``released to science''.

It is rumoured that Zeldovich was traded for a fuel pump !

Zeldovich defended his PhD in 1936 and, years later, reminiscenced of:

``the happy times when permission to defend [a PhD] was granted to people with no higher education''.

The fact that Zeldovich was primarily self-taught enormously influenced his style of doing research and also teaching.

During the 1930's, Zeldovich extensively worked on nuclear physics writing seminal papers demonstrating the possibility of controlled fission chain reactions among uranium isotopes. Soon the USSR was in the grips of WW II.

According to Andrei Sakharov:
``*from the very beginning of Soviet work on the atomic (and later thermonuclear) problem, Zeldovich was at the very epicenter of events.'*
His role there was completely exceptional."

Zeldovich's earlier work on combustion paved the way for creating the internal ballistics of solid-fuel rockets which formed the basis of the Soviet missile program during the `great patriotic war' and after.

(Sadly, much of Zeldovich's work during this period remains classified.)

Zeldovich moved to Astrophysics in 1962 when he was nearing 50 !

Almost immediately he started making pioneering contributions in key areas:

Black hole physics,

Dark matter,

Quantum field theory in curved space-time,

The cosmological constant problem,

Topological defects,

CMB: Sunyaev-Zeldovich effect,

Large scale structure: *Zeldovich approximation,* etc.

For an ansatz to be successful it should embrace within its fold the behaviour of a reasonably large class of Dark Energy models.

An ansatz involving only 2 free parameters can describe, DE whose equation of state evolves moderately with redshift. It is quite clear that these simple fits cannot be used to rule out models with rapidly evolving $w(z)$.

To accommodate models with a fast transition in the EOS one might try:

$$1. \quad w(z) = w_i + \frac{w_f - w_i}{1 + \exp(\frac{z - z_t}{\Delta})} \ ,$$

[Bassett *et al* MNRAS 336, 1217, 2002; also see Corasaniti *et al*, PRL 90, 091303, 2003]

$$2. \quad w(z) = -\frac{1 + \tanh\left[(z - z_t)\Delta\right]}{2} \ .$$

[Shafieloo *et al*, PRD 80, 101301, 2009; see also Ishida *et al*, 2008]

One should note that while increasing the number of parameters increases the accuracy of reconstruction of the \`best fit', this is often accompanied by severe degeneracies which limit the utility of introducing a large number of free parameters.

Model independent reconstruction of Dark Energy

Let us define Dark Energy through the Einstein equations:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G \left(\sum_a T_{\mu\nu}^{(a)} + T_{\mu\nu}^{DE} \right)$$

then, in a spatially flat universe

$$H^2 = \frac{8\pi G}{3} \left(\sum_a \rho_a + \rho_{DE} \right) \ , \quad \frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \left(\sum_a (\rho_a + 3p_a) + \rho_{DE} + 3p_{DE} \right)$$

If one neglects radiation, then these equations give $(q \equiv -\ddot{a}/aH^2)$

$$\rho_{\rm DE} = \frac{3H^2}{8\pi G}(1 - \Omega_{\rm m}) \ , \qquad p_{\rm DE} = \frac{H^2}{4\pi G}(q - \frac{1}{2}) \ ,$$

From where we obtain the effective equation of state of dark energy

$$w \equiv p_{DE}/\rho_{DE} = \frac{2q(x) - 1}{3(1 - \Omega_m(x))} \equiv \frac{\frac{2x}{3}\frac{d \log H}{dx} - 1}{1 - (H_0/H)^2 \Omega_{0m} x^3} \ , \ x = 1 + z \ .$$

Example 2. The Chaplygin gas which has $p = -A/\rho$ can be described by a minimally coupled scalar field with the potential

$$V(\phi) = \frac{\sqrt{A}}{2}\left(\cosh(2\sqrt{6\pi G}\phi) + \frac{1}{\cosh(2\sqrt{6\pi G}\phi)}\right) ,$$

and associated with the Lagrangian density $\mathcal{L} = \frac{1}{2}\dot{\phi}^2 - V(\phi)$.

[Kamenschik, Moschella and Pasquier, PLB, 2001, Gorini et al, PRD 2005]

However the Chaplygin gas can also be modeled completely differently using a scalar field with the Born-Infeld kinetic term:

$$\mathcal{L} = -V_0\sqrt{1 - \phi_{,\mu}\phi^{,\mu}} .$$

[Bilic et al, PLB 2002, Frolov et al, PLB 2002]

This once more illustrates the fact that the equation of state $w(z)$ does not uniquely define an underlying field-theoretic model !

Good news ! CMB determines $\Omega_{0m}h^2$ to great accuracy in LCDM cosmology:

$$\Omega_{0m}h^2 = 0.1426 \pm 0.0025$$

To test LCDM: [A] Determine Omh^2 from

$$Omh^2(z_i, z_j) = \frac{h^2(z_i) - h^2(z_j)}{(1+z_i)^3 - (1+z_j)^3}, \text{ where } h(z) = \frac{H(z)}{100\text{km/sec/Mpc}}$$

[B] check whether $Omh^2 = \Omega_{0m}h^2 = 0.1426 \pm 0.0025$

If [B] holds then dark energy = cosmological constant, if not surprise !

Independent measurements of H(z) are available at 3 redshifts: $z = 0, 0.57, 2.34$

$H(z=0) = 70.6 \pm 3.3,\ H(z=0.57) = 92.4 \pm 4.5,\ H(z=2.34) = 222 \pm 7$ km/sec/Mpc

[Efstathiou, 2014, Samshia et al, 2013 Delubac et al, 2014]
BOSS DR11

Leading to $Omh^2(z_1, z_2) = 0.124 \pm 0.045,\ Omh^2(z_1, z_3) = 0.122 \pm 0.010,$
$Omh^2(z_2, z_3) = 0.122 \pm 0.012$

Result: the model independent value $Omh^2 \simeq 0.122$ is stable and is in tension with the LCDM based value $Omh^2|_{\rm LCDM} \simeq 0.14$!

Tension with LCDM is at over 2σ !

[VS, Shafieloo, Starobinsky 2014]

BAO:

TABLE I: BAO data from different surveys. The two high z Lyα points have a distinct character to the low redshift data, and the data are often divided into two sets– low redshift Galaxy BAO data and high redshift Lyα data.

Source	z	D_V/r_d	σ	D_M/r_d	σ	D_H/r_d	σ	$r_{\rm off}$
6dFGS	0.106	3.047	0.137	0	0	0	0	0
SDSS-MGS	0.15	4.480	0.168	0	0	0	0	0
BOSS-LOWz	0.32	8.467	0.167	0	0	0	0	0
BOSS-CMASS	0.57	0	0	14.945	0.210	20.75	0.73	−0.52
LyaFauto	2.34	0	0	37.675	2.171	9.18	0.28	−0.43
LyaF-QSOcross	2.36	0	0	36.288	1.344	9.00	0.30	−0.39

$$r_d = \frac{1}{H_0}\int_{z_d}^{\infty}\frac{c_s(z)dz}{h(z)}; \quad c_s(z) = \frac{c}{\sqrt{3}\sqrt{1+0.75\frac{\Omega_{0b}h^2}{\Omega_{0\gamma}h^2(1+z)}}} \quad (1)$$

$$D_H(z) = \frac{c}{H_0 h(z)} \quad (2)$$

$$D_M(z) = \frac{c}{H_0}\int_0^z \frac{dz}{h(z)} \quad (3)$$

$$D_V(z) = \left[z D_H(z) D_M^2(z)\right]^{1/3} \quad (4)$$

where $z_d = 1059.68$ is the Planck value for the drag redshift

SNIa: Union2.1 SNe Ia dataset consisting of 580 SNe between z = 0.01 and 1.4.

CMB data is encapsulated into two parameters, the shift parameter R, and the angular scale of the sound horizon at last scattering, l_A.

$$R = \sqrt{\Omega_{0m} H_0^2} D_A(z_\star)/c$$
$$l_A = \pi D_A(z_\star)/r_s(z_\star) \ ,$$

$D_A(z)$ is the comoving angular diameter distance and $r_s(z)$ the comoving sound horizon at redshift z. $z_\star$ is the redshift for which the optical depth is unity.

$$R = 1.7382 \pm 0.0088, \ l_A = 301.63 \pm 0.15 \text{ at } z_\star = 1089.9$$

Ade et al. arXiv:1502.01590

$$S = M^3 \left[\int_{\text{bulk}} (\mathcal{R} - 2\Lambda) - 2 \int_{\text{brane}} K \right] + \int_{\text{brane}} \left(m^2 R - 2\sigma \right) + \int_{\text{brane}} L\left(g_{\mu\nu}, \phi \right) ,$$

This action leads to the following effective equation on the brane:

$$G_{\mu\nu} + \left(\frac{\Lambda_{\text{RS}}}{b+1} \right) g_{\mu\nu} = \left(\frac{b}{b+1} \right) \frac{1}{m^2} T_{\mu\nu} + \left(\frac{1}{b+1} \right) \left[\frac{1}{M^6} Q_{\mu\nu} - \mathcal{C}_{\mu\nu} \right] , \qquad (1)$$

Weyl fluid -- projection of the 5D weyl tensor from the bulk to the brane.

where

$$b = \frac{\sigma \ell}{3M^3} , \qquad \ell = \frac{2m^2}{M^3} , \qquad \Lambda_{\text{RS}} = \frac{\Lambda}{2} + \frac{\sigma^2}{3M^6} \qquad (2)$$

are convenient parameters, and

$$Q_{\mu\nu} = \frac{1}{3} E E_{\mu\nu} - E_{\mu\lambda} E^{\lambda}{}_{\nu} + \frac{1}{2} \left(E_{\rho\lambda} E^{\rho\lambda} - \frac{1}{3} E^2 \right) g_{\mu\nu} , \qquad (3)$$

$$E_{\mu\nu} \equiv m^2 G_{\mu\nu} - T_{\mu\nu} , \qquad E = E^{\mu}{}_{\mu} . \qquad (4)$$

The tensor $\mathcal{C}_{\mu\nu}$ is not freely specifiable on the brane, but is related to the tensor $Q_{\mu\nu}$ through the conservation equation

$$\nabla^{\mu} \left(Q_{\mu\nu} - M^6 \mathcal{C}_{\mu\nu} \right) = 0 ,$$

Scalar perturbations of the induced metric on the brane:

$$ds^2 = -(1+2\Phi)dt^2 + a^2(1-2\Psi)\gamma_{ij}dx^i dx^j$$

$$\delta T_{(\lambda)}{}^{\mu}{}_{\nu} = \begin{pmatrix} -\delta\rho_\lambda\,, & -\nabla_i v_\lambda \\ \dfrac{\nabla^i v_\lambda}{a^2}\,, & \delta p_\lambda \delta^i{}_j + \dfrac{\zeta_\lambda{}^i{}_j}{a^2} \end{pmatrix}\,,$$

Perturbed matter stress-energy tensor

$\delta\rho_\lambda$, δp_λ, v_λ, and $\zeta_{\lambda\, ij} = \left(\nabla_i\nabla_j - \frac{1}{3}\gamma_{ij}\nabla^2\right)\zeta_\lambda$ describe scalar perturbations.

In a similar way we introduce scalar perturbations $\delta\rho_{\mathcal{C}}$, $v_{\mathcal{C}}$, and $\delta\pi_{\mathcal{C}}$ of the traceless tensor $\mathcal{C}_{\mu\nu}$:

$$m^2\delta\mathcal{C}^{\mu}{}_{\nu} = \begin{pmatrix} -\delta\rho_{\mathcal{C}}\,, & -\nabla_i v_{\mathcal{C}} \\ \dfrac{\nabla^i v_{\mathcal{C}}}{a^2}\,, & \dfrac{\delta\rho_{\mathcal{C}}}{3}\delta^i{}_j + \dfrac{\delta\pi^i{}_j}{a^2} \end{pmatrix}\,,$$

Weyl fluid

where $\delta\pi_{ij} = \left(\nabla_i\nabla_j - \frac{1}{3}\gamma_{ij}\nabla^2\right)\delta\pi_{\mathcal{C}}$.

In order to close the system of equations one needs a relationship between $\delta\pi_{\mathcal{C}}$ and $\delta\rho_{\mathcal{C}}$

Such a relation has been derived in the limit of a marginally closed braneworld:

$$\delta\pi_{\mathcal{C}} = \frac{3a^4}{2k^4}\left[\delta\ddot{\rho}_{\mathcal{C}} + \left(9H - \frac{\dot{H}}{H}\right)\delta\dot{\rho}_{\mathcal{C}} + \left(20H^2 + \frac{k^2}{3a^2}\right)\delta\rho_{\mathcal{C}}\right]$$

Viznyuk, Shtanov and Sahni, PRD 89, 083523 (2014)

On sub-Hubble scales, when $k \gg aH$, this eqn reduces to $\delta\pi_{\mathcal{C}} \approx \frac{a^2}{2k^2}\delta\rho_{\mathcal{C}}$

which is the quasi-static approximation of Koyaama and Maartens (2016).

To investigate perturbations of a multi-component fluid, we introduce convenient variables

$$\delta_\lambda \equiv \frac{\delta\rho_\lambda}{\rho_\lambda + p_\lambda}, \qquad V_\lambda \equiv \frac{v_\lambda}{\rho_\lambda + p_\lambda}$$

$$\delta_{\mathcal{C}} \equiv \frac{\delta\rho_{\mathcal{C}}}{\rho_r + p_r} = \frac{3\delta\rho_{\mathcal{C}}}{4\rho_r}, \qquad V_{\mathcal{C}} \equiv \frac{v_{\mathcal{C}}}{\rho_r + p_r} = \frac{3v_{\mathcal{C}}}{4\rho_r}.$$

Velocity potentials

ΛCDM has a remarkable property: Higher derivatives of the expansion factor can be written solely in terms of the second derivative !

This is unlikely to be the case in other DE models…..

Taylor expanding a(t) we get: $$(1+z)^{-1} := \frac{a(t)}{a_0} = 1 + \sum_{n=1}^{\infty} \frac{A_n(t_0)}{n!}[H_0(t-t_0)]^n$$

where $A_n := \frac{a^{(n)}}{aH^n}$, $a^{(n)}$ is the nth derivative of $a(t)$.

For $n > 3$, $A_n = f(\ddot{a})$ in ΛCDM ! For instance: $(q = -\ddot{a}/aH^2)$

$$A_4 = 1 - 3(1+q), \quad A_5 = 1 + 2(1+q)(4+3q), \ etc.$$

One can exploit this relationship to construct a hierarchy of null tests of ΛCDM :

$$S_4 = A_4 + 3(1+q), \ S_5 = A_5 - 2(1+q)(4+3q), \ etc.$$

The Statefinder hierarchy, S_n, stays pegged to unity in ΛCDM : $S_n\big|_{\Lambda CDM} = 1$

During the entire course of cosmic expansion !!

In other words, $S_n\big|_{\Lambda CDM} = 1$ defines a hierarchy of null tests for ΛCDM !

$S_n \neq 1$, in models with evolving DE.

Arabsalmani & Sahni -- PRD 83, 043501 (2011).

$$S_4 = A_4 + 3(1+q), \quad \text{where } A_n := \frac{a^{(n)}}{aH^n}, \quad a^{(n)} \text{ is the nth derivative of } a(t).$$

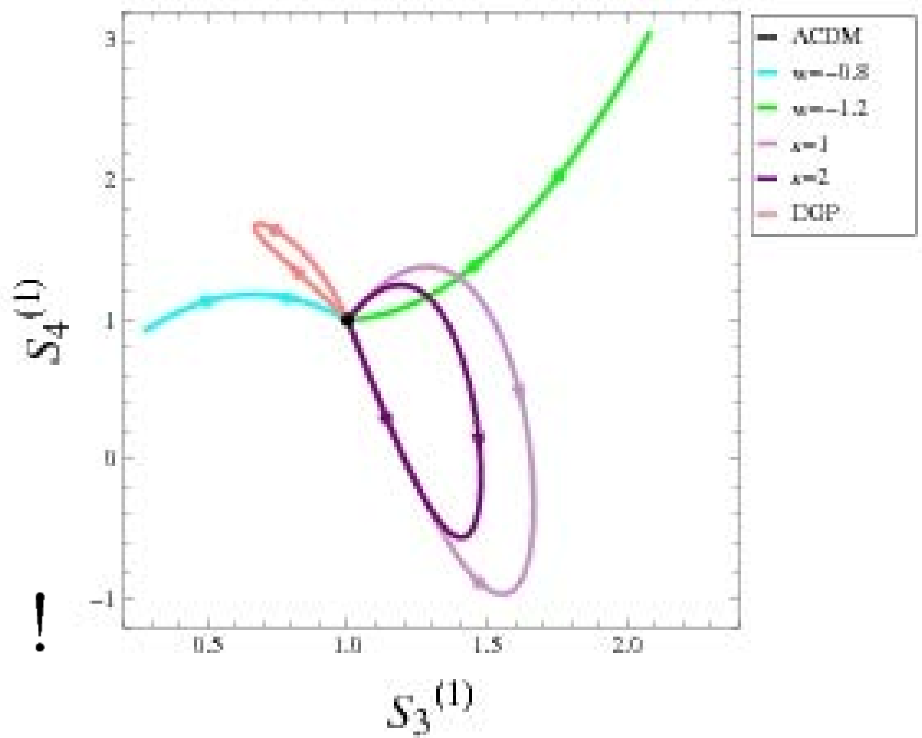
$$S_3 = \dddot{a}\,/aH^3, \;\; q = -\ddot{a}/aH^2$$

The diagnostic pair $\{S_4, S_3\}$ can easily differentiate between various DE models including:
Chaplygin gas,
Quintessence, Phantom, DGP and

ΛCDM !

$$S_3 = 1, \;\; S_4 = 1, \;\; \text{for } \Lambda\text{CDM}$$

$$\Rightarrow \Lambda\text{CDM is a fixed point !}$$



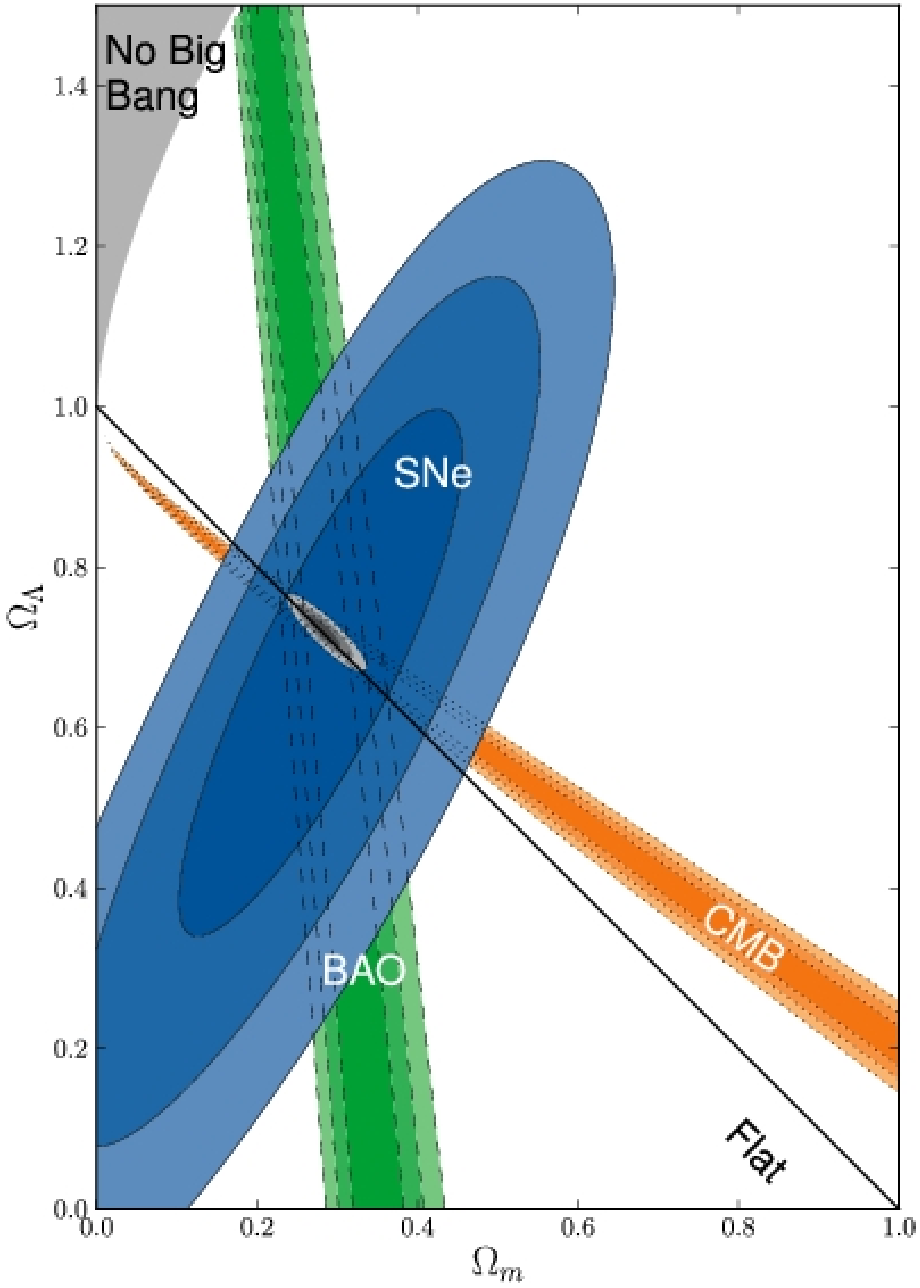
Advantage: the S_n parameters contain information about (n-1)th derivative of H(z). Therefore higher order Statefinder's contain lots of theoretical information about the evolution of DE .

Disadvantage: for this very reason higher order S_n may be difficult to determine observationally while the Om diagnostic will prove useful in the near future….

[Arabsalmani and VS, PRD 2011]

The Cosmological Constant provides excellent agreement with observations !

$$\Omega_\Lambda \simeq 2/3,\ \Omega_m \simeq 1/3$$



Suzuki et al arXiv:1105.3470

Exploding Stars Point to a Universal Repulsive Force

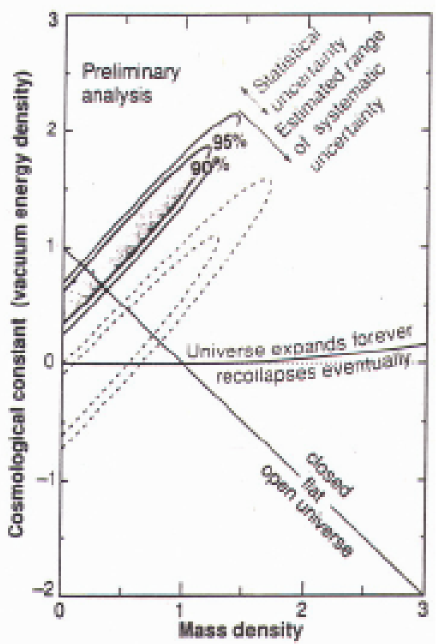
By now, even newspaper readers with a casual interest in astronomy may have heard the unsettling message delivered by distant, exploding stars called supernovae: The universe will likely expand infinitely, growing ever more tenuous. Now a new batch of supernovae has lent support to a strange picture of just what the universe is made of. A preliminary analysis may provide the first strong evidence that the universe could be permeated by a large-scale repulsive force. The reservoir of energy fueling that force could be anything from a quantum-mechanical shimmer in empty space, called the cosmological constant, to even more exotic possibilities that go by names like X-matter and quintessence.

At the meeting of the American Astronomical Society in Washington, D.C., earlier this month, Saul Perlmutter of Lawrence Berkeley National Laboratory in Berkeley, California, announced that he and an international team of observers have now studied a total of 40 far-off supernovae, using them as beacons to judge how the cosmic expansion rate has changed over time. Not only did the results support the earlier evidence that the expansion rate has slowed too little for gravity ever to bring it to a stop; they also hinted that something is nudging the expansion along. If they hold up, says Perlmutter, "that would introduce important evidence that there is a cosmological constant."

"It would be a magical discovery," adds Michael Turner of the University of Chicago. "What it means is that there is some form of energy we don't understand." Other observers had already found signs that the universe contains far less mass than the mainstream theory of the big bang predicts, which left open the possibility that some form of energy in empty space could be making up the deficit. The cosmological constant—also called lambda—is a longtime candidate for serving as this energy reservoir. But the new

was sparked when a fleck of the primordia vacuum underwent a chance fluctuation thai filled it with something much like a colossally intense cosmological constant. This "scalar," or directionless, field drove the patch into an exponential growth spurt. As the patch expanded and cooled, energy from the scalar field fed an explosion of material particles: The material universe was born—"creating everything from nothing," as the theory's creator, Alan Guth of the Massachusetts Institute of Technology, puts it.

During the exponential growth spurt, inflation would have ironed out any primordial



What the stars show. A preliminary analysis of 40 distant supernovae, reported by the Supernova Cosmology Project, offers strong evidence for an energy density in empty space, if space is "flat." The green regions indicate statistical uncertainties; the dashed lines show the preliminary estimates (now being refined) if all the systematic uncertainties added up in one direction.

Dawn of Dark Energy

Type Ia Supernovae used as standard candles.

Science 30 January 1998

[Based on Perlmutter, et al., Ap J (2009); also see Riess, et. al. Astron. J (2008)]

Model independent reconstruction of Dark Energy: two approaches

[A] Study cosmic expansion using geometrical parameters:

$$H = \dot{a}/a, \;\; q = -\ddot{a}/aH^2, \;\; r = \dddot{a}\,/aH^3, \;\; \text{etc.}$$

which arise in the Taylor expansion the expansion factor

$$a(t) = a(t_0) + \dot{a}\big|_0 (t - t_0) + \frac{\ddot{a}\big|_0}{2}(t - t_0)^2 + \frac{\dddot{a}\,\big|_0}{6}(t - t_0)^3 + \cdots$$

[B] Study cosmic expansion via physical parameters such as $(x = 1 + z)$

$$\rho_{\rm DE} = \frac{3H^2}{8\pi G} - \rho_m \;, \qquad w \equiv p_{DE}/\rho_{DE} = \frac{2q(x) - 1}{3(1 - \Omega_m(x))}$$

Note that the definition of physical parameters, $\rho_{\rm DE}$, w , is based on the validity of general relativity. Consequently, $\rho_{\rm DE}$, $w(z)$, as defined above could show very unusual behaviour in modified gravity theories !